Modelling of Reinforced Earth Beds Using Lumped Parameter Models: A Review

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ABSTRACT
This paper deals with a review of the models developed for the study of response of reinforced earth beds. The reinforcement can be in the form of geosynthetic or stone columns. The emphasis has been given to the models proposed in the lumped parameter mode. The paper states the details about the development of various lumped parameter models starting from very basic Winkler model to more sophisticated ones. Although, various models take into account various properties of the earth beds and geosynthetic to be more realistic, most of these lack in some or other aspect while modeling the true representation of the soil – foundation system. In the present paper, these deficits have also been pointed out with respect to various models.

1. INTRODUCTION

To understand the behaviour of the soil – foundation system, various models have been developed for many decades. With the passage of time, these models were modified to present more realistic representation of the soil – foundation system. These models adopt discrete as well as continuum approach. In discrete models, the soil is assumed to be represented by discrete elements however, as the name suggests, in continuum models the soil is considered to be a continuum. Although the continuum models present the system in a more realistic manner, however, discrete models are quite widely used due to its simplicity. Winkler (1867) presented a model in which the soil was idealized by a set of discrete, linear and independent springs. The main limitation of this model is that the displacement occurs immediately under the loaded area and outside this region the displacements are zero.This model was modified by various research workers to overcome its limitations. Some of the modified models include Filonenko – Borodich model (1945), Hetenyi's model (1946), Pasternak model (1954) etc.

The rapid development of infrastructure led to an increased demand for land. Scarcity of good land and their exorbitant cost especially in metropolitan cities forced the developers to construct in areas having very poor supporting capacity giving birth to the development of a new area called ground engineering. Out of several options to improve upon the ground conditions (improved bearing capacity and reduced settlement), reinforcement of weak soils by using either metallic or geosynthetics inclusions or with the help of stone columns have become very common and popular with the Geotechnical engineers. The above mentioned basic models were modified to represent the earth, reinforced with geosynthetics or stone columns. Some of these models include Madhav and Poorooshasb (1988, 1989), Ghosh and Madhav (1994 a, b, c), Shukla and Chandra (1994 a, b, c), Yin (1997 a, b; 2000 a, b), Maheshwari et al. (2004 a, b, c; 2005), Deb et al. (2005; 2007a, b, c) etc. Although various aspects related to reinforced earth beds have been incorporated in these models, however, some of the aspects like slippage, visco – elastic behaviour, parameter estimation etc need to be studied in detail. In the subsequent sections, detail discussion has been carried out on these models and their development.

2. BASIC LUMPED PARAMETER MODELS FOR IDEALIZING SOIL MASS

Various research workers developed many lumped parameter models for representing the behavior of soil. Selvadurai (1979) has summarized all these in detail. However, some of these have been presented below.

2.1 Winkler Model

Winkler (1867) proposed an idealized model of soil media which assumes that the deflection, w, of the soil medium at any point on the surface is directly proportional to the stress, q, applied at that point and independent of stresses applied at other locations, i.e.,

$$q(x,y) = k\,w(x,y) \qquad [1]$$

where, k is the modulus of subgrade reaction having units of stress per unit length. Winkler's idealization of soil mass comprises of a system of mutually independent springs having spring constant as k. An important feature of this model is that the displacement occurs only under the loaded area. For various types of loading, the surface displacements of Winkler model have been shown in Fig. 1. The limitations of this model include discontinuity in deformation profile and further this can not distinguish between an infinitely rigid load and a uniform flexible load (Figures 1 (c) and (d)).

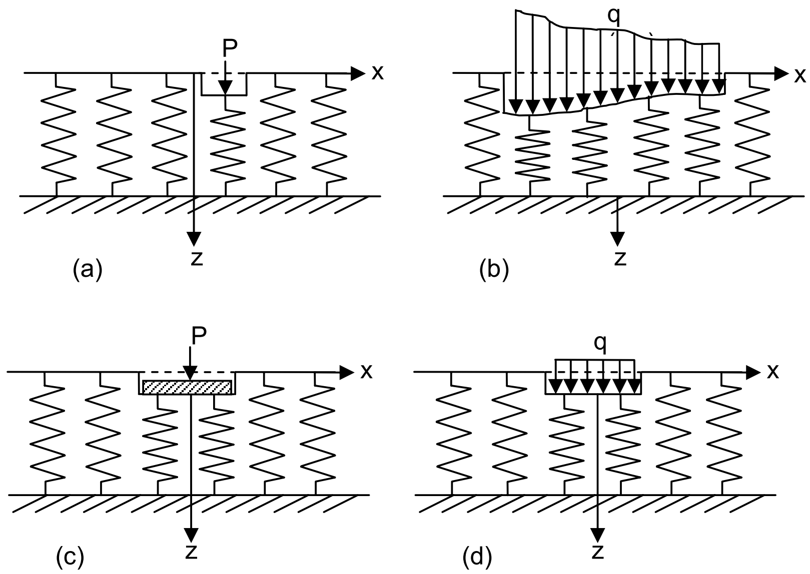

Figure 1. Surface displacements of Winkler model due to (a) a concentrated load, (b) a non – uniform load, (c) a rigid load, (d) a uniform flexible load.

2.2 Filonenko – Borodich Model

To remove the inherent deficiency of Winkler model in depicting the continuous behaviour of real soil masses, a model was proposed by Filonenko – Borodich (1940, 1945). This model provides continuity between individual spring elements in Winkler model by connecting them to a thin elastic membrane under a constant tension T (Fig. 2). The equilibrium of membrane – spring system yields the surface deflection of the soil medium due to a pressure, q as:

$$q(x,y) = k\,w(x,y) - T\,\nabla^2 w(x,y) \qquad [2]$$

where, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, is Laplace's differential operator in rectangular Cartesian coordinates. The two elastic constants, k and T characterize the soil model. Typical surface deflection profiles due to concentrated, flexible and rigid external loads have been depicted in Fig. 2.

2.3 Hetenyi's Model

Hetényi (1946) proposed a model in which the interaction between the independent spring elements is established by incorporating an imaginary elastic plate (in 3 – D problems) or an elastic beam (in 2 – D problems). The surface deflection due to a pressure, q is given by:

$$q(x, y) = k\, w(x, y) - D\nabla^4 w(x, y) \quad [3]$$

where, $D = \frac{E_p h^3}{12\left(1 - \nu_p^2\right)}$ is the flexural rigidity of the plate. h be the thickness of plate and E_p and ν_p are the elastic constants for the plate material.

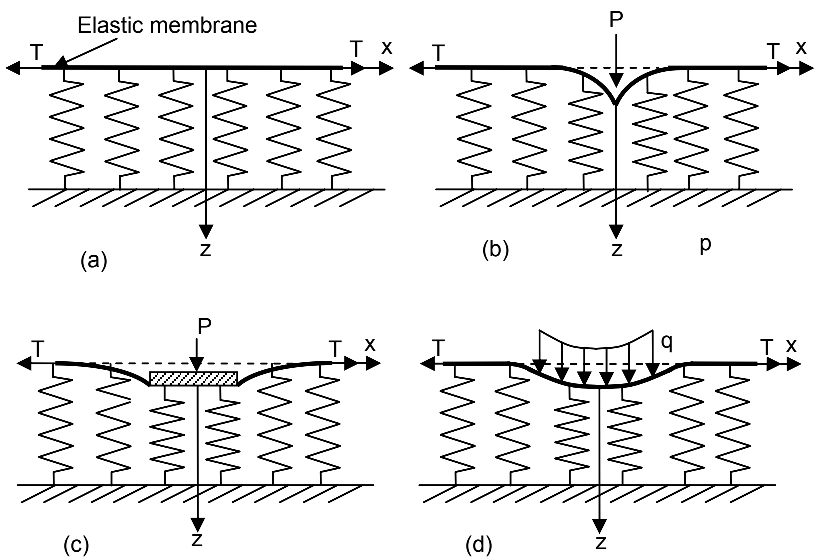

Figure 2. Surface displacements of Filonenko – Borodich model (a) due to (b) concentrated load, (c) rigid load, (d) uniform flexible load.

2.4 Pasternak Model

Pasternak (1954) presented a model which assumes shear interaction between the spring elements which was accomplished by connecting these spring elements to a layer of incompressible vertical elements deforming only in transverse shear (Figure 3). A free body diagram of an element of shear layer has been depicted in Figure 3. Force equilibrium in the z – direction yields the following relation:

$$q(x, y) = k\, w(x, y) - G\nabla^2 w(x, y) \quad [4]$$

where, G is shear modulus of the shear layer which is considered to be isotropic in x, y plane. Filonenko – Borodich model (1940, 1945), Hetényi (1946) and Pasternak model (1954) reduces to Winkler model (1867) as the respective parameters, T, D and G tend to zero.

2.5 Rheological Models

To represent the time dependent behaviour of soil mass, these models are employed. These models make use of various arrangements of Hookean spring (k) with Newtonian dashpot (η). When these two are arranged in parallel, the resulting model is called as a Kelvin model and for a series arrangement; the model is named as a Maxwell model. These have been presented in Fig. 4.

3. MODELING OF GEOSYNTHETIC REINFORCED EARTH BEDS

Some of the basic models as presented above have been employed by various research workers to model the geosynthetic reinforced earth beds subjected to various types of loading. Madhav and Poorooshasb (1988) proposed a 3-parameter mechanical model for geosynthetic-reinforced granular fill-soft soil system as shown in Fig. 5. In this model, the geosynthetic reinforcement was represented by a rough membrane. The granular fill and the soft subgrade were idealized by Pasternak shear layer and a layer of Winkler springs respectively. The results at small displacement indicated the effect of granular fill to be more and significant than that of the membrane in reducing the settlements of the reinforced soft soil system. The effects of providing geosynthetic reinforcement were felt at higher loads or on soft soils.

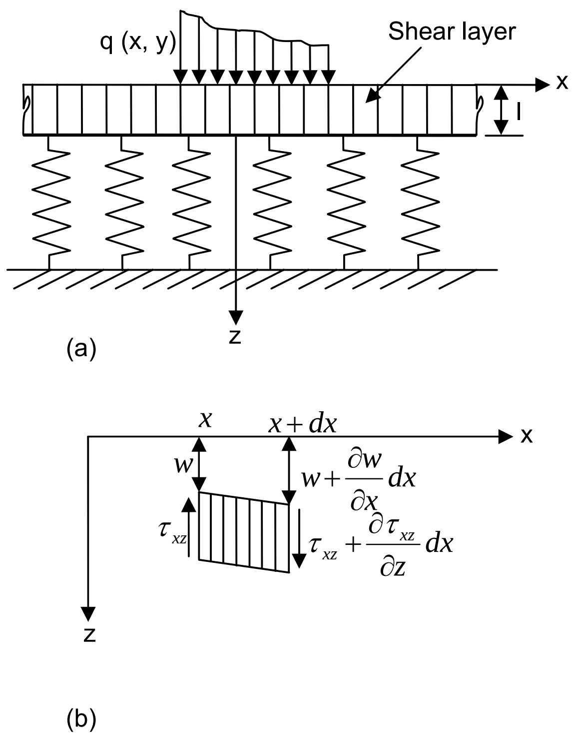

Figure 3. The Pasternak model. (a) basic model, (b) stresses in the shear layer.

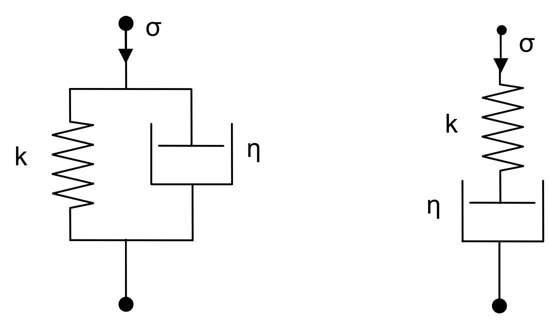

Figure 4. Kelvin and Maxwell models.

This model was linear in nature and was not able to represent the behaviour of soft soil, i.e., soil having visco – elastic behaviour. Further, the effect of membrane on confining stress in granular fill was also not considered in the analysis. In view of later, Madhav and Poorooshasb (1989) investigated the effect of membrane in increasing the confining stress in the granular material with a consequent increase of the shear modulus (G) with distance with Pasternak type foundation model. Analysis of a simple Pasternak type foundation with variable G showed a significant reduction in the overall settlements as well as in the differential settlements due to the increased shear moduli. It was pointed out that extending the reinforcement beyond 2B (width of the footing) on either side of the centre of footing has less effect on settlements within the loaded region.

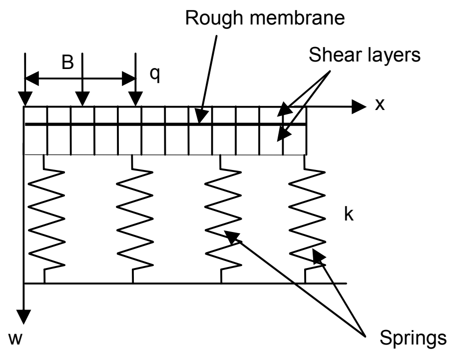

Figure 5. Proposed Model for granular fill – geofabric – soft soil system (Madhav and Poorooshasb 1988)

Ghosh and Madhav (1994 a) proposed a new three-parameter model incorporating the nonlinearities in the load-settlement and the shear stress-strain responses, respectively, of soft clay and granular fill for the analysis of a reinforced granular fill-soft soil system. The complete load-settlement response of the strip footing obtained brought out the contribution of the various parameters of the system (Fig. 6). The improvements in the settlement behaviour were significant with respect to stiffness of granular fill, when the soil was softer, and with respect to interfacial friction, when the fill material was less stiff. The improvement in settlement response due to reinforcement was of the same order, and over and above the effect of granular fill.

Ghosh and Madhav (1994 b) developed a mathematical model for the analysis of a reinforced foundation bed by incorporating the confinement effect of a single layer of reinforcement. It was quantified in terms of the average increase in confining pressure due to the reinforcement from which modified shear stiffnesses of the granular soil surrounding the reinforcement were obtained. The confinement effect was more pronounced when the shear stiffness of the granular fill was large. It was found that the modified shear stiffness below the centre of the footing may increase by two to five times the initial values of shear stiffness. Further, mechanics of the rough membrane element with the assumption of horizontal shear stress transfer at the soil/reinforcement interfaces were explained and formulated by Ghosh and Madhav (1994 c). The model was generalized by incorporating nonlinear response of the soft soil and of the granular fill under plane strain loading conditions. Parametric studies indicated that reinforcement while in tension spreads the load over a larger area, leading to a reduction in the settlement beneath the footing.

Shukla and Chandra (1994 a) modified the model proposed by Madhav and Poorooshab (1988) to study the effect of prestressing the geosynthetic reinforcement on the settlement behaviour of geosynthetic-reinforced granular fill-soft soil system. They concluded that the settlement of a reinforced foundation bed caused mainly due to sagging of the reinforcing elements could be improved significantly by prestressing the same. Shukla and Chandra (1994 b) further incorporated the compressibility of the granular fill by attaching a layer of Winkler springs to the Pasternak shear layer. Time dependent behaviour was studied by Shukla and Chandra (1994c).

Yin (1997 a) presented a one-dimensional mathematical model for geosynthetic reinforced granular fills over soft soils subjected to a vertical surcharge load. This model was based on the assumption of a Pasternak shear layer and a deformation compatibility condition was introduced. This condition eliminated two shear stress parameters and made it possible to include the stiffness of the geosynthetic in the model. Yin (1997 b) extended his model for geosynthetic reinforced granular fills over soft soils using a nonlinear constitutive model for granular fill and a nonlinear spring model for soft soil.

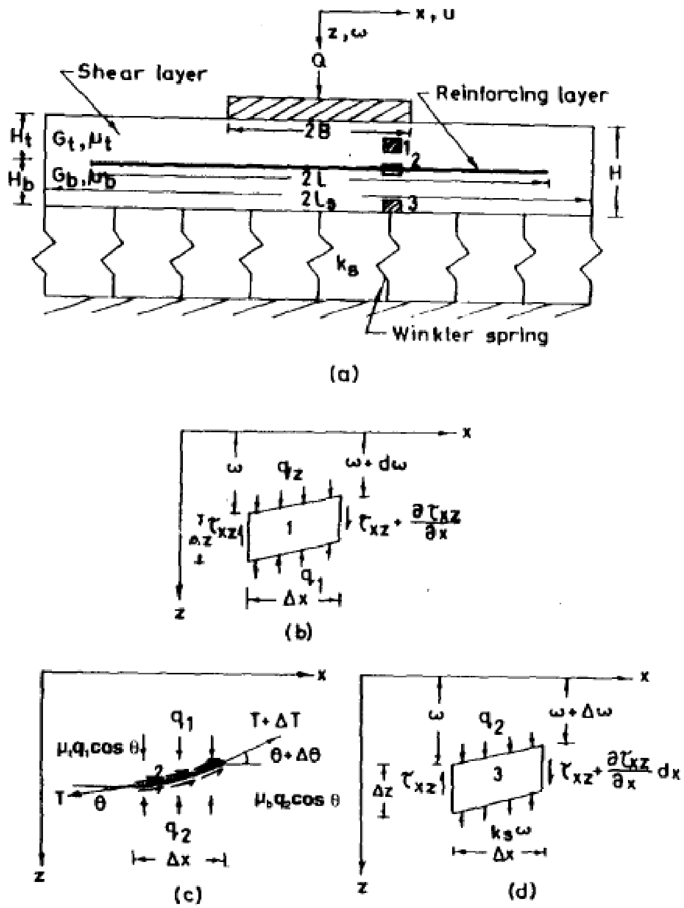

Figure 6. Definition sketch for geosynthetic reinforced earth beds (Ghosh and Madhav 1994a)

Yin (2000 a) gave an analytical solution for a point load on a reinforced Timoshenko beam on elastic foundation. The results from TB model (Timoshenko beam on elastic foundation) were compared with results from the FE model (finite element model). Yin (2000 b) suggested a method for obtaining closed-form solutions for a reinforced Timoshenko beam on an elastic foundation subjected to any pressure loading (Fig. 7). The results showed that the settlement without consideration of the shear stiffness of geosynthetic was larger than that with geosynthetic shear stiffness, while the tension was opposite. The settlement and rotation decreased while bending moment, tension and shear force increased as tension modulus of the geosynthetic increased.

In all the above models, the bending stiffness of the reinforcement was not taken into consideration. In view of this, Maheshwari et al. (2004a) proposed a model which considered the bending stiffness of the geosynthetic layer as shown in Fig. 8. This model was a modification to Hetenyi's model. It finds application in the analysis of combined footings and being simple, can be used efficiently. However, because of its linear nature while representing the soil, it has limitation in its predictions.

All the models presented above analyzed the soil – foundation system subjected to static loading conditions. Maheshwari et al. (2004 b) proposed a model for the analysis of such systems under moving loads as depicted in Fig. 9. The results from the study revealed that the response of infinite beam is greatly affected by magnitude and velocity of applied load and relative compressibility of the granular fill. The geosynthetic layer was considered as extensible as well as inextensible in these studies. This model was modified by Maheshwari et al. (2004c, 2005) for taking into account of the separation of the beam from the ground surface. That is, the analysis was conducted for the tensionless foundation.

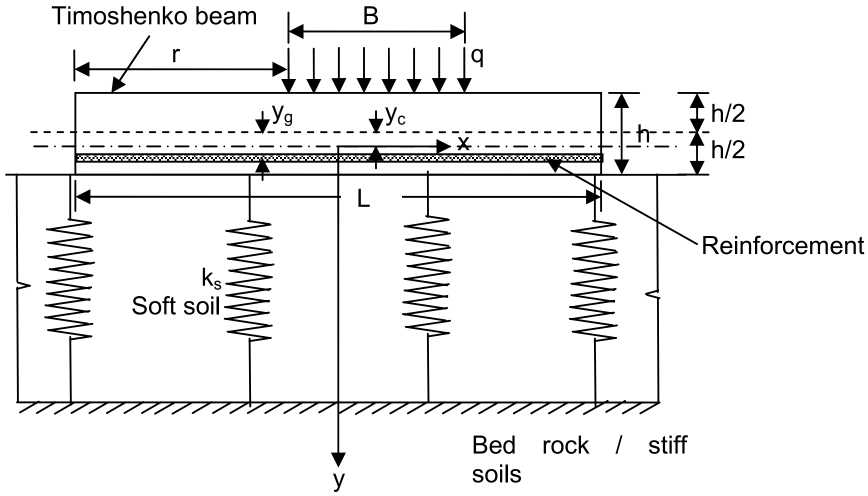

Figure 7. Schematic diagram for one – dimensional foundation model as proposed by Yin (2000b)

The models developed by previous research workers for a single layer of geosynthetic reinforcement were modified by Deb et al. (2005, 2007a) by considering the multiple reinforcement layers. Significant reduction of the settlement was observed as a result of the use of the multilayer geosynthetic reinforcement system. The non-linear model was found to give more accurate results than the linear model in the case of very soft soil. The top reinforcement layer was subjected to maximum mobilised tension at the centre of the loaded area. The settlement value was found to reduce by an amount nearly the same as the increase of modular ratio and was independent of the number of geosynthetic layers.

Apart from reinforcing the soil only with geosynthetic layers, stone columns are also quite widely employed for the same. Deb et al. (2007b) developed a mechanical model to predict the behaviour of a geosynthetic-reinforced granular fill over soft soil improved with stone columns. The saturated soft soil has been idealized by Kelvin–Voight model to represent its consolidation behaviour. The stone columns were idealized by stiffer springs. Pasternak shear layer and rough elastic membrane represented the granular fill and geosynthetic reinforcement layer, respectively (Fig. 10). The nonlinear behaviour of the granular fill and the soft soil was considered. Effect of consolidation of the soft soil due to inclusion of the stone columns has also been included in the model. Plane strain conditions were considered for the loading and reinforced foundation soil system. The advantage of using geosynthetic reinforcement was highlighted. Results indicated that inclusion of the geosynthetic layer effectively reduces the settlement. Nonlinearity in the behaviour of the soft soil and the granular fill was found to reduce due to the use of geosynthetic reinforcement layer. In all the models the creep for the geosynthetic was not considered. In view of this a model was proposed by Deb et al. (2007c) to consider the rheological behaviour of the geosynthetic.

4. CONCLUDING REMARKS

In the present paper, a detailed review of lumped parameter models for the modeling and analysis of geosynthetic reinforced earth beds has been discussed. It is quite evident that some of the advance models take care of more realistic behaviour of soil – foundation system. Although these models, being simple in nature, are used quite widely, however these have many limitations as far as the response of the soil – foundation system is concerned as compared to those from models based on continuum approach. The first difficulty associated with the use of these models is the estimation of the model parameters like modulus of subgrade reaction, shear modulus of granular layer etc. Although an average value of these parameters on the basis of limited test data is adopted, however, these do not provide a realistic picture. For some situation, the estimation of associated parameters becomes quite cumbersome and difficult. Further, the idealization of soil by lumped parameter models does not provide

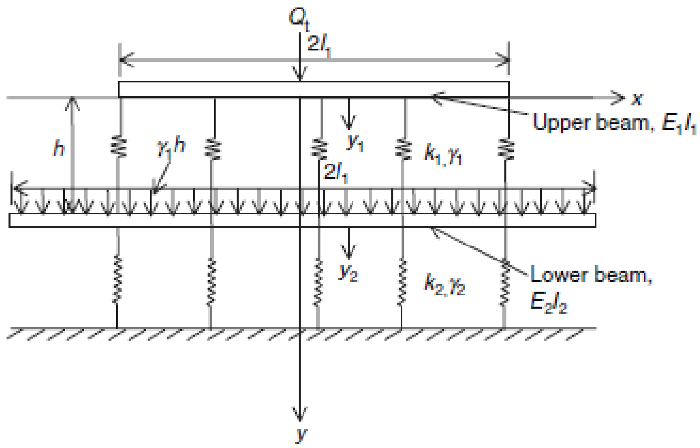

Figure 8. Schematic diagram of proposed model by Maheshwari et al. (2004a).

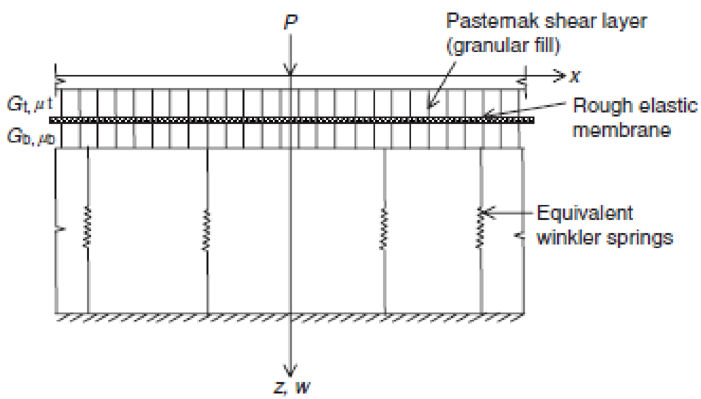

Figure 9. Schematic diagram of proposed model by Maheshwari et al. (2004b, c) for moving loads on infinite beams

a realistic picture as it is a continuum and therefore, models based on continuum approach depict more realistic picture. As such, quite advance lumped parameter models have been developed for the analysis of soil – foundation system. However, some aspects need further attention. Some of these include modeling of slippage between geosynthetic and neighbouring soil, non – linearity and time dependent behaviour, modeling of disturbance while installing reinforcing elements etc.

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