# EuroGeo4 Paper number 108 EFFECT OF SOIL REINFORCEMENT ON THE RESPONSE OF VERTICALLY VIBRATING FOUNDATIONS USING DIFFERENT ISOLATION SYSTEMS

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**Abstract:** The dynamic loads generated in the machine are transmitted to the supporting foundation and underlying soil. Different vibration isolation systems could be placed between the machine and the foundation block to reduce the transmission of vibration. Also, soil reinforcement may be used to improve the performance of foundations even subjected to dynamic loading. The vibration response of a machine-foundation-soil system is defined by its natural frequency and its amplitude of vibration. Therefore, these are the two most important parameters to be determined in designing the machine foundation. In this paper, the effect of soil reinforcement on the dynamic responses of a machine foundation under vertical vibration is investigated. The results demonstrate the extent of efficiency of soil reinforcement in improving the performance of machine foundations for different vibration isolation systems.

Keywords: dynamic loads, soil reinforcement, design method.

### **INTRODUCTION**

The design of foundations supporting machinery that may be subjected to dynamic loadings has been a subject of considerable interest over the past few decades (Gazetas 1983, Sienkiewicz and Wilczynski 1994). Vibrations of machine foundations induce elastic waves in soil which may destructively affect surrounding buildings and their effects range from serious disturbances of working conditions for sensitive devices and people to visible structural damage. The vibration responses of a machine-foundation-soil system can be characterized by its natural frequency and its amplitude of vibration. The vibration response limits of machine and machine foundation were established to meet the stability and serviceability requirements, and/or to minimize any disturbance to the neighborhood and surroundings. Different vibration isolation systems such as viscous isolator springs and/or inertia block are now used to minimize the vibration transmission. The details of isolators and the design procedure for foundation with vibration isolation system can be found in any machine foundation's handbook. The efficient design of isolation system depends on the intended purpose, the type of dynamic loads, the mass ratio between machine and machine foundation and the dynamic characteristics of foundation. The influence of various parameters of the isolator mounting system on the impact force transmitted to soil was investigated, and a number of dynamic models were introduced for one-mass and two-mass foundations with springs and dampers (Novak 1983; El Hifnawy and Novak 1984; Chehab and El Naggar 2003, 2004; Wang and Dong 2006). The dynamic soil properties can be changed to improve the performance of machine foundations; therefore, Soil reinforcement is considered as a potentially advantageous technique to enhance the performance of foundation systems under dynamic loads (El Naggar and Wei 1997, Das 1998). This paper investigates the effect of geogrid reinforcement on the dynamic response of a machine foundation using different vibration isolation systems under a harmonic vertical load. The results reveal the extent of efficiency of reinforcement in improving the dynamic response of vertically vibrating machine foundation using different isolation systems. For the design of foundations in which vibrations are taken into account, it is necessary to establish criteria to specify if the design is satisfactory or not. In this study, a guideline that was compiled by Richart (1962) is used for allowable vertical vibration amplitudes as shown in Figure 1.

## DYNAMICS OF MACHINE-FOUNDATION-SOIL SYSTEM

For the design of foundations supporting machinery that may acts as a source of vibration, the resonant frequency of machine-foundation-soil system and the amplitude of vibration must be determined. A variety of theoretical methods have been proposed for determination of the dynamic response of such foundations. A versatile technique for analysis of foundation vibrations is a mass-spring-dashpot system (i.e., a lumped parameter system). The foundation system is usually modeled either as a one or two-mass system depending on the configuration of the foundation. Large machines are often placed on a foundation block. Vibration isolation is imposed to separate the dynamical system from its environment by adding a suspension and damping elements (often called vibration isolators) as shown in Figure 2a, or by adding an inertia block, a large mass (usually a block of cast concrete), directly attached to the machine (Figure 2b).

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Figure 1. Allowable vertical vibration amplitudes (Richart 1962)



**Figure 2.** Machine foundation with different configuration of vibration isolation system a) Direct isolator support, b) Isolator supported inertia block

On the other hand, the key parameters influencing the dynamic response of machine-foundation-soil system are the vibration mode, the geometry, the rigidity and embedment of the foundation, and the properties of the supporting soil deposit and backfill. So, the soil foundation should be represented by spring-dashpot system tacking these factors into account.

# MATHAMATICAL MODEL

Machine foundations with vibration isolation systems under vertical harmonic loading can be modeled using two-mass model as illustrated in Figure 2. The equations of motion can be represented in matrix form as

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1 & -c_1 \\ -c_1 & c_1 + c_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f_{(t)} \\ 0 \end{bmatrix}$$
[1]

where  $m_1$  is the mass of the machine (Figure 2a) or the mass of the machine plus inertia block (Figure 2(b)). The mass  $m_2$  is the mass of the foundation block and all the parts attached to it. The stiffness and damping constants  $k_1$  and  $c_1$  represent the stiffness and damping of the spring and dashpot of vibration isolation system, respectively. Similarly,  $k_2$  and  $c_2$  are the stiffness and damping coefficients representing the soil foundation as shown in Figure 3. The machine and foundation block responses are  $x_1(t)$  and  $x_2(t)$ . Finally, the vertical harmonic force is  $f(t) = f_0 \sin \omega t$  where  $\omega$  is the natural frequency for vertical vibration.



Figure 3. Two mass foundation model

In this study, a computer program based on a numerical method (i.e. central difference method) is encoded to solve the differential equations and obtain displacement and velocity time histories. The force transmitted through the vibration isolation system can then be calculated as

$$f_{1(t)} = k_1(x_1 - x_2) + c_1(\dot{x}_1 - \dot{x}_2)$$
[2]  
and the force transmitted through the soil is  
$$f_{2(t)} = k_2 x_2 + c_2 \dot{x}_2$$
[3]

Then, the force transmissibility,  $T_F$ , is defined as the ratio of the amplitudes of the force transmitted to the soil support to the force f (t) applied to the machine

$$T_{F} = \frac{f_{2(t)}}{f_{(t)}}$$
[4]

### ISOLATOR STIFFNESS AND DAMPING

The most important element in vibration isolation is the isolation material itself, which in turn depends on its characteristics and performance. There are lots of materials used for isolation such as springs, elastomers (natural and synthetic), PVC cork and fiber filled pads, composite materials and natural and synthetic foams. All these materials have many advantages in different types of applications. Typical vibration isolators employ a helical spring to provide stiffness, and an elastomeric layer (such as neoprene) to provide some damping. Other types use a solid elastomeric element for both the stiffness and the damping. The practical characteristics of all these isolation materials can be measured and confirmed by laboratory tests. The stiffness and damping constants of isolation systems are usually supplied by the manufacturer.

## FOUNDATION STIFFNESS AND DAMPING

The effect of embedment on the vertical vibration of a circular footing was shown by Kaldjian (1969) using an elastic finite element solution. It could increase not only the equivalent spring constant but also the equivalent damping provided through the soil layers adjacent to the foundation sides. For embedded foundation in a homogenous semi infinite medium, expressions were developed for frequency dependent stiffness  $(k_2)$  and frequency dependent damping coefficients  $(c_2)$  as follows (wolf 1985)

$$k_2 = GBk_{11}$$
 [5]  
 $c_2 = GBc_{11}$  [6]

where G is the dynamic shear modulus of a supporting medium; B is the half-width of the rectangular foundation base;  $V_s$  is the shear wave velocity and  $k_{11}$  and  $c_{11}$  are the normalized stiffness and damping coefficients, respectively.

The normalized functions  $k_{11}$  and  $c_{11}$  depend upon the following dimensionless parameters

$$k_{11} = f_1\left(a_0, \lambda, \overline{h}, \overline{g}, \overline{\rho}, \nu, \beta, \beta_s\right)$$
<sup>[7]</sup>

 $c_{11} = f_2 \left( a_0, \lambda, \overline{h}, \overline{g}, \overline{\rho}, \nu, \beta, \beta_s \right)$ [8]

where  $a_0 = \omega B \sqrt{\rho_B'}$  is the dimensionless frequency ratio;  $\lambda = L_B'$  is the aspect ratio;  $\overline{h} = h_B'$  is the embedment

ratio;  $\overline{g} = \frac{G_s}{G}$  is the shear-modulus ratio;  $\overline{\rho} = \frac{\rho_s}{\rho}$  is the density ratio;  $\rho$  and  $\rho_s$  are the densities of a supporting

medium and backfill, respectively;  $G_s$  is the shear modulus of the backfill; v is the Poisson's ratio of supporting medium;

L is the half-length of the rectangular base and at last  $\beta$  and  $\beta_s$  are the hysteretic damping ratios of the supporting material and backfill, respectively.

The algebraic expressions for calculating the dynamic lumped parameters  $k_{11}$  and  $c_{11}$  of a rectangular rigid embedded foundation are given in the Appendix, where the material damping of supporting medium has been incorporated by means of the correspondence principle (Veletsos and Verbic 1973, Lysmer 1980).

#### SOIL REINFORCEMENT

The use of geosynthetics to improve the bearing capacity and settlement performance of shallow foundations has proven to be a cost-effective foundation system. A Reinforced Soil Foundation (RSF) consists of one or more layers of a geosynthetic reinforcement and controlled fill placed below a conventional spread footing to create a composite material with improved performance characteristics. There are a number of factors that may influence the performance of an RSF, including: 1) type of reinforcement; 2) number of reinforcing layers in the zone of influence, N; 3) depth below the footing to the first layer of reinforcement, u/B; 4) spacing between reinforcing layers, h/B; 5) width of reinforcement layers, b/B; 6) total depth of reinforcement, d/B ;7) type of imported loads; and 8) type and placement of the fill. Some of these parameters for geogrid reinforced sand with respect to the foundation are shown in Figure 4.





Soil reinforcement improves the dynamic properties of the soil (Montanelli et al., 2003; Shuwang et al. 2004) and the response of footings to harmonic loading (El Naggar and Wei, 1997). El Naggar and Wei (1997) found that the effect of reinforcing the side layer (backfill) was more pronounced than reinforcing the bottom layer (supporting soil). Although, it was shown that for the same maximum depth of reinforcement, the shear modulus increases with the number of layers in place, but further study is needed to determine the optimum values of the influencing parameters mentioned above. In order to include the effect of reinforcement in the theoretical analysis, the modified shear modulus,  $G_{sr}$ , and equivalent density,  $\rho_{sr}$ , of the reinforced soil are substituted into equations 6 and 7.

## PARAMETRIC STUDY

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A comprehensive parametric study is conducted by increasing the shear modulus and density of backfill to investigate the effect of soil reinforcement on the dynamic response of machine-foundation-soil system. The shear modulus ratio

$$(R = \frac{G_{sr}}{G_s})$$
 is defined to represent the effect of increasing the shear modulus on the dynamic properties of soil foundation.

As the shear modulus of RSF increase with the number of reinforcement layers, its equivalent density may change subsequently. The geogrid could have a density between 1.2 to 1.7 megagrams per cubic meter ( $Mg/m^3$ ), which is too close to the common values of soil density. Therefore, the equivalent density of reinforced backfill may change slightly so that it can be assumed to remain constant.

The different mass ratios  $(m_1/m_2)$  are representatives of different configurations of isolated foundations and may vary from 0.5 to about 3 for large machine. The dynamic response of machine- foundation-soil system may be sensitive to the initial value of isolator stiffness and damping. The damping of the isolators is usually selected about 5–10% of the foundation damping, i.e.  $c_1/c_2 = 0.05-0.1$ , and the vibration isolation system can be useful as long as the stiffness  $k_1$  is less than about 0.3 (Chehab and El Nagger, 2003). The stiffness and damping of the reinforced foundation  $(k_{2r}, c_{2r})$  are normalized by the stiffness and damping of the foundation  $(k_2, c_2)$ . The maximum vibration amplitudes of the machine and the foundation block in the two-mass reinforced foundation system  $(A_{1r}, A_{2r})$  are normalized by their maximum vibration amplitudes in the two mass foundation system  $(A_1, A_2)$ . Similarly, the frequencies of the two-mass reinforced foundation system  $(f_{1r}, f_{2r})$  are normalized by the relevant frequencies in a two-mass foundation system  $(f_{1r}, f_{2r})$ . Also, the maximum force transmitted to the reinforced soil foundation is normalized by the force transmitted to the supporting medium  $(T_{Fr}/T_F)$ .

## **RESULTS AND DISCUSSION**

The responses of machine and machine foundation to the vertical harmonic forces with different dimensionless frequency ratio of  $a_0=2.04$  and 3.45 (by assuming different frequencies for vertical load) for different mass ratio  $m_2/m_1=1$  and 3 are demonstrated in the Figures 5 and 6, respectively. The effect of backfill reinforcement is considered by varying the shear modulus ratio R from 1 to 10 that alters the dynamic characteristics of soil foundation. As the shear modulus of backfill increase, the maximum amplitude and frequency of machine and foundation block are calculated and the maximum amplitude of force transmitted to the supporting soil is determined. Figures 5a and 5b present the variation of machine and machine foundation amplitudes with their relevant frequencies, respectively, for two different dimensionless frequency ratios as the shear modulus of backfill increases in 10 steps. The amplitude of machine increase slightly as the shear modulus of the reinforced soil increases relative to the native soil as can be noted from Figure 5a.

Figure 5b shows that the amplitude of machine foundation can decrease significantly (up to 40 %) and strongly depends on the frequency of applied force or dimensionless frequency ratio up to the point that further soil improvement has an adverse effect of increasing the amplitude of foundation block. The frequency of machine foundation increases significantly (up to 70%) and depending on the dimensionless frequency ratio, its adverse effect may be reduced. Figure 5c presents the variation of force transmissibility with shear modulus ratio. The frequency of harmonic vertical load may enhance the unfavorable effect of increasing the force transmissibility considerably (from 20% to 70%).

Similar observations can be obtained from Figure 6 that display the effect of soil reinforcement on the dynamic response of machine foundation system for  $m_2/m_1=3$ . Generally, the variations of the machine and foundation amplitudes of the foundation on RSF with the frequency are likely to be the same as the mass ratio increases. Considering Figure 6a compared to Figure 5a, it can be noted that the adverse effect of increasing the frequency of machine enhances slightly by increasing the mass ratio  $m_2/m_1$ . Also, comparing Figures 6b and 5b, it is clear that the beneficial effect of decreasing the foundation amplitude has only an improvement for lower mass ratio and the frequency of machine foundation remains almost constant as the mass ratio increases. At last, considering Figure 6c compared to Figure 5c, it can be noted that the force transmissibility of the dynamic system increase as the mass ratio increases.







**Figures 5 and 6.** The effect of soil reinforcement on the response of machine-foundation-soil system under harmonic vertical loads with two different dimensionless frequency ratios (i.e.  $a_0=2.04$  and  $a_0=3.45$ )

## CONCLUSIONS

The effect of soil reinforcement on the dynamic response of vertically vibrating foundation is investigated. The results reveal that the amplitude of machine slightly increases by reinforcing the backfill and the amplitude of foundation block can decrease considerably and is strongly dependent to the frequency of vertical load. On the other hand, the frequency of machine increases as the shear modulus of the reinforced soil increases, and depending on the frequency of harmonic force, the frequency of the machine foundation may be increased significantly up to 70 %. The response of foundation block is considered to be more sensitive to the soil reinforcement than the response of machine increases slightly. For higher mass ratio, the beneficial effect of decreasing the amplitude of machine foundation reduces and its frequency has a slight improvement. The soil reinforcement has an adverse effect of increasing the force transmitted to the soil that can be more significant for higher mass ratio. In regards to Figure 1, both amplitude and frequency of machine and machine foundation control the design criteria. Also, the force transmissibility is a key parameter regarding the bearing capacity of supporting medium. Therefore, the effect of soil reinforcement and vibration isolation systems on all these factors should be considered to obtain an efficient design for a vertically vibrating foundation.

Acknowledgements: The first author would like to express his profound gratitude to Mr. Reza Sedghamiz, who, helped him to edit the paper.

## APPENDIX

Functions k<sub>11</sub> and c<sub>11</sub> (Novak et al. 1978, Lysmer 1980, Wolf and Somaini 1986, Paris and Kausel 1988)

$$k_{11} = \frac{1}{1-\upsilon} (3.1\lambda^{0.75} + 1.6)k_{11}^*(a_0, \beta) + \overline{ghk_{11}}(a_1^L, \beta_s)$$

$$c_{11} = \frac{1}{1-\upsilon} (3.1\lambda^{0.75} + 1.6)c_{11}^*(a_0, \beta) + \overline{gh}\gamma_1 c_{11}^L(a_1^L, \beta_s)$$
[10]

$$k_{11}^{*}(a_{0},\beta) = k_{11}^{0}(a_{0}) - a_{0}\beta c_{11}^{0}(a_{0})$$
[12]

$$c_{11}^{*}(a_{0},\beta) = k_{11}^{0}(a_{0}) - a_{0}\beta c_{11}^{0}(a_{0})$$
[13]

$$k_{11}^0(a_0) = 1 - \frac{0.14a_0^2}{1 + 0.218a_0^2}$$
[14]

$$c_{11}^{0}(a_{0}) = \frac{0.065a_{0}^{2}}{1 + 0.218a_{0}^{2}} + 0.4(\lambda - 1)^{2/3} + 0.9$$
[15]

$$k_{11}^{L}(a_{1}^{L},\beta_{s}) = \operatorname{Re}\left[2\pi(1+2i\beta_{s})a_{1}^{*}\frac{K_{1}(a_{1}^{*})}{K_{0}(a_{1}^{*})}\right]$$
[16]

$$c_{11}^{L}(a_{1}^{L},\beta_{s}) = \operatorname{Im}\left[2\pi(1+2i\beta_{s})^{1/2}i\frac{K_{1}(a_{1}^{*})}{K_{0}(a_{1}^{*})}\right]$$
[17]

$$a_{1}^{*} = ia_{1}^{L} (1 + 2i\beta_{s})^{-1/2}, \qquad a_{1}^{L} = \gamma_{1}a_{0}$$

$$(4\lambda \ \overline{\rho}) \qquad (18)$$

$$\gamma_1 = \left(\frac{4\lambda}{\pi} \frac{\rho}{\overline{g}}\right), \qquad i = \sqrt{-1}$$
[19]

In which  $K_0(z), K_1(z)$  = modified Bessel functions of the second kind of the complex argument z.

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