COMPARISON OF AN ANALYTICAL SOLUTION FOR MULTI-STEP REINFORCED SOIL SLOPES WITH CONVENTIONAL NUMERICAL METHODS

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Abstract: This paper focuses on the verification of an analytical solution, which has been developed for the purposes of this study and concerns multi-step reinforced-soil slopes that are subjected to static and seismic loading. This verification is accomplished through the comparison of the analytical solution with i) conventional methods (such as Bishop) and ii) elasto-plastic finite element stress analysis (FESA).

The analytical solution is based on the kinematic theorem of limit analysis and on the quasi-static approach and concerns homogenous cohesionless soils that are expected to deform plastically, following the Coulomb yield criterion. A computer program based on this methodology has been designed (developed in Delphi), which defines: a) the tensile strength, length and spacing of the reinforcement that are necessary to prevent failure and b) the critical yield acceleration of the reinforced slopes.

The results of this limit analysis are imported into a representative 2D limit equilibrium slope stability program, and the Safety Factors of the models are calculated.

The results of the limit analysis are also imported into a 2D elasto-plastic finite element stress analysis program. Through this analysis, the Strength Reduction Factors of the models and the developed stresses along the surface of the reinforcement layers are calculated. The axial forces are compared with the necessary tensile strength of the reinforcement that derives from the analytical solution.

Finally, one step and multi-step Reinforced Soil Slopes (RSS) models with the same height and mechanical characteristics are designed and compared with the use of the analytical solution, the limit equilibrium analysis and the elasto-plastic FESA, and the advantages of the multi-step RSS are provided.

Keywords: limit equilibrium, stability analysis, failure mechanism, reinforced slope, seismic design, finite element

INTRODUCTION

Soil is ideal for use in construction, since it is a relatively inexpensive and abundant construction material. Moreover, soil is capable of providing very high strength in compression, but virtually no strength in tension. Like other construction materials with limited strength, soil can be reinforced with materials such as strips, grids, sheets, rods and fibres in order to form a composite material that has better mechanical characteristics. The need to reinforce soil is even higher in seismically active areas and this application has expanded in the last 20 years.

The stability of a reinforced slope is related to the geometrical and mechanical characteristics of the construction. Slope height and inclination along with static and seismic loading conditions determine the amount of the reinforcement that is necessary to prevent failure. Slopes with small height and gentle inclination demand relatively low reinforcement. On the other hand, high and steep slopes are vulnerable to earthquake loading and therefore require long reinforcement with higher tensile strength. In addition, surface water flows over a larger area with higher velocities along high and steep slopes, leading to erosion. For these reasons, there is a requirement for another approach to the design and construction of high slopes in order to limit these problems.

The design of high reinforced-soil slopes with steps has many advantages. Soil weight loading and inertial force induced by the earthquake loading are reduced and therefore the necessary tensile strength and length of the reinforcement is lower. In addition, the surface water collects on the step berm, limiting the potential for surface erosion. Reinforced, steep embankments with irregular, curved ground plan have been constructed for the Egnatia Highway in Greece.

The analysis method for multi-step reinforced slopes that is presented here and emphasizes the former advantages is based on the kinematic theorem of limit analysis and on the quasi-static approach and concerns homogenous cohesionless soils that are expected to deform plastically, following the Coulomb yield criterion.

ANALYSIS METHODS

Various methods of analysis have been used in order to define the behaviour of reinforced slopes subjected to seismic and static loading.

The limit analysis method considers the stress-strain relationship of the soil in an idealized manner. This idealization (expressed by the flow rule) establishes the limit theorems on which limit analysis is based. Within this framework, the approach is rigorous and the techniques are in some cases much simpler than those of limit equilibrium. The plastic limit theorems of Drucker & Prager (1952) can be employed to obtain upper and lower boundaries of the collapse load for stability problems.

Conventional analysis methods such as Bishop and Janbu are also applied to obtain solutions in soil stability problems. The stability of slip surfaces is analyzed using slice limit equilibrium methods and the safety factors of circular or non circular failure surfaces in soil slopes are evaluated.

The finite element analysis method is a very comprehensive approach and stress-strain analysis can be performed. Moreover, the soil-reinforcement interface can be taken into account and the developed stresses along the surface of the reinforcement can be defined. This can only be achieved by performing finite element stress analysis and not by analytical solutions. However, an analytical solution is an accurate, closed form solution with small numerical cost and therefore can be used as a fast tool for the design of soil structures.

The analysis method presented in the current study is based on the kinematic theorem of limit analysis and on the quasi-static approach.

Kinematic theorem of limit analysis

The kinematic theorem of limit analysis is based on the upper bound theory of plasticity and states that the slope will collapse if the rate of work undertaken due to external loads and body forces exceeds the energy dissipation rate in any kinematically admissible failure mechanism.

$$\int_V \sigma_{ij}^* \varepsilon_{ij}^* dV^3 \int_S T_i \, v_i dS + \int_V X_i v_i^* dV, \; i,j=1,2,3 \tag{1}$$

Where ε_{ij}* is the strain rate in a kinematically admissible velocity field, σ_{ij}* is the stress tensor associated with εij*, velocity vi*= vi on boundary S (given kinematic boundary condition), Xi is the vector of body forces (unit weight and the distributed quasi-static inertial force), and S and V are the loaded boundary and the volume, respectively.

In this study, pore water pressure and potential liquefaction are not considered. The rate of external work is due to soil weight and inertia force induced by the seismic loading and the only contribution to the energy dissipation is provided by the reinforcement. In addition, it is assumed that the energy dissipation is performed only by the tensile strength of the geosynthetics, while resistance to shear, bending and compression are ignored.

Quasi-static approach and seismic coefficient

According to the quasi-static approach, a static force with horizontal direction represents the seismic influence on the failure soil mass. This force is estimated by the product of seismic intensity coefficient and weight of the potential sliding soil mass. This approach is a widely accepted method, despite the fact that it neglects the acceleration history.

The evaluation of the seismic coefficient can be accomplished with various empirical predictive relations based on the seismotectonic environment of each region.

Analytical solution of multi-step reinforced slopes

According to this method, a high and steep reinforced slope is divided into more slopes with smaller height and scaled inclination. For example, a slope with 50 meters height and average inclination 3:2 is divided into 5 slopes with 10 meters height each and scaled inclination as shown in Figure 1.

In order to determine the amount of the reinforcement that is necessary to prevent failure, different failure modes are examined and the most critical ones are used for the final design. The failure mechanism presented here is the plane failure mechanism.

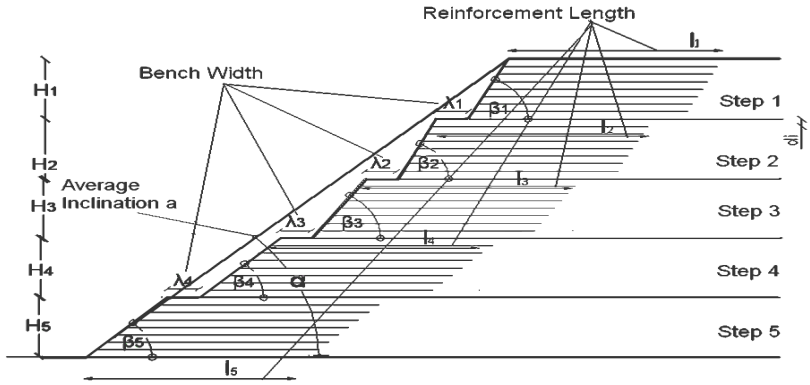

Figure 1. Multi-Step reinforced slope.

In the plane failure mechanism, it is assumed that the reinforced soil mass translates as a rigid body with velocity V (Figure 2). The height "H" of the slope and angle "Ω" that the failure plane forms from the horizontal specifies the failure mechanism. The main goal is to determine the critical value of Ω (for a slope with given height) and therefore define the critical failure mechanism. Once the critical failure mechanism is defined, the amount of the required reinforcement can be calculated.

The plane failure mode is applied twice, once in order to ensure that the tensile strength and length of the reinforcement are adequate against local stability for each step separately (Figure 2) and then against global stability for the whole slope (Figure 3). It is necessary that both failure modes are applied, since the failure mode that concerns local stability gives critical results for the upper layers of the reinforcement and the failure mode for global stability for the lower layers.

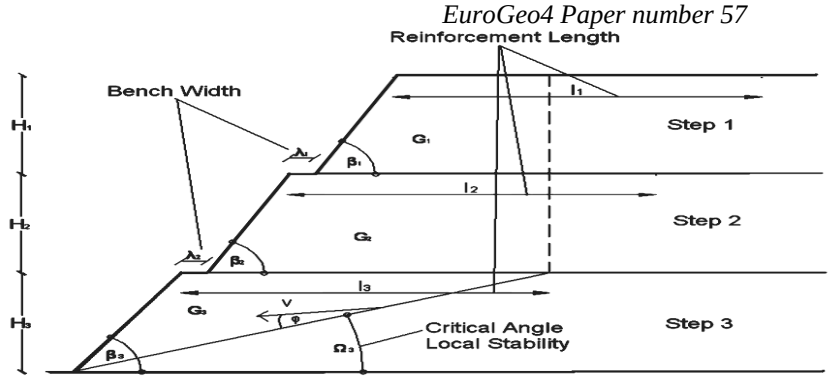

Figure 2. Local stability failure mode, Step 3

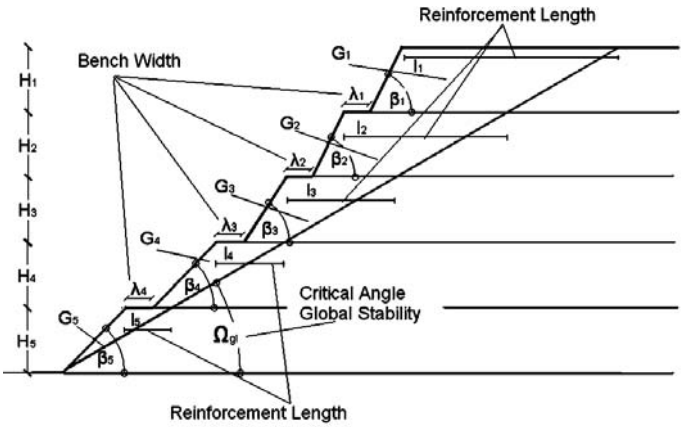

Figure 3. Global stability failure mode

In particular, the rate of external work done by soil weight and inertial force is:

$$\dot{W} = (G_1 + G_2 + \ldots + G_i) V \sin(\Omega - \varphi) + k_h (G_1 + G_2 + \ldots + G_i) V \cos(\Omega - \varphi) \tag{2}$$

Where G_1, G_2… G_i indicates the weight of the soil wedge for each step, with different expressions for local and global failure modes and k_h is the horizontal seismic coefficient. In the current study, examples with five and one step slopes are examined.

According to Ausilio et al (2000), the energy dissipation is

$$\dot{D} = V \cos(\Omega - \varphi) \sum_{i=1}^{n} T_i \tag{3}$$

Moreover, Ling et al (1997) suggested that the tensile strength (T_i) can be calculated approximately by the following equation:

$$T_i = K \gamma z_i d_i \tag{4}$$

Where K, represents the total reinforcement in a normalized form with the following expression:

$$K = \frac{\sum_{i=1}^{n} T_i}{(1/2) \gamma H^2} \tag{5}$$

and d_i is the tributary area of layer i.

Eq. (3) owing to Eq. (4) becomes:

$$\dot{D} = \frac{1}{2} V \cos(\Omega - \varphi) K \gamma H^2 \tag{6}$$

Equating the rate of external work due to the soil weight and inertial force to the energy dissipation, the total reinforcement in a normalized form (K) can be calculated for Local and Global failure mode:

a) Local Stability:

$$K_i = \frac{2(G_1+\ldots+G_i)\tan(\Omega_i-\varphi_i)+2k_h(G_1+\ldots+G_i)}{\gamma_i H_i^2} \tag{7}$$

By substituting the maximum values of K_i to Equation 4, the maximum necessary tensile strength can be calculated for each step. Moreover, the required length of reinforcement for each step is:

$$l_i = \frac{H_i \sin(\beta_i - \Omega_i)}{\sin(\beta_i)\sin(\Omega_i)} \tag{8}$$

(The required length cannot be lower than 0.7H)

b) Global Stability

$$K_{gl} = \frac{2(G_1+G_2+\ldots+G_i)\tan(\Omega_{gl}-\varphi_{gl})+2k_h(G_1+G_2+\ldots+G_i)}{\gamma(H_1+H_2+\ldots+H_i)^2} \tag{9}$$

The required tensile strength of reinforcement for each layer derives by substituting again the maximum value of K_{gl} to Equation 5.

The required length of reinforcement for global stability can be calculated by the following equation:

$$l_i = \frac{H_i + H_{i+1} + \ldots + H_n}{\tan(\Omega_i)} - \frac{H_i}{\tan(\beta_i)} - \frac{H_{i+1}}{\tan(\beta_{i+1})} - \ldots - \frac{H_n}{\tan(\beta_n)} - \lambda_i - \lambda_{i+1} \ldots - \lambda_{n-1} \tag{10}$$

(The required length cannot be lower than 0.7H)

For the final design, the maximum values of the required tensile strength and length for each layer are chosen. As mentioned before, local stability calculations result in longer reinforcement with larger required tensile strength for the upper layers and global stability calculations are more critical for the lower layers. By applying both analyses, the multi step reinforced soil slope can be designed in a more accurate and safe way.

Similarly, the critical acceleration factor can be obtained:

a) Local Stability:

$$k_{yi} = \frac{K_i \gamma_i H_i^2 - 2(G_1+G_2+\ldots+G_i)\tan(\Omega_i-\varphi_i)}{2(G_1+G_2+\ldots+G_i)} \tag{11}$$

b) Global Stability:

$$k_{ygl} = \frac{K_{gl}\gamma_i(H_i+H_{i+1}+\ldots+H_n)^2 - 2(G_1+G_2+\ldots+G_i)\tan(\Omega_{gl}-\varphi_i)}{2(G_1+G_2+\ldots+G_i)} \tag{12}$$

Where k_y, is the yield acceleration factor.

The minimum value of k_y is the critical yield acceleration of the slope.

Similarly, equations for design with static loading can be obtained:

a) Local Stability:

$$K_i = \frac{2(FS_i)(G_1+G_2+\ldots+G_i)\tan(\Omega_i-\varphi_i)}{\gamma_i H_i^2} \tag{13}$$

b) Global Stability:

$$K_{gl} = \frac{2(FS_{gl})(G_1+G_2+\ldots+G_i)\tan(\Omega_{gl}-\varphi_{gl})}{\gamma_{gl}(H_1+H_2+\ldots+H_i)^2} \tag{14}$$

Where F.S. is the desirable Factor of Safety.

SOFTWARE IMPLEMENTATION

For the implementation of the former methodology, a computer programme has been designed. This software has been developed in Borland Delphi and has so far the following features.

- Calculation of the Horizontal Peak Ground Acceleration (HPGA) based on empirical predictive relations. The user can import the appropriate predictive relation based on the seismotectonic characteristics of each region. An independent value can also be chosen.
- Dynamic analysis for local and global failure mode. Plane failure, direct sliding and log spiral mechanism are available. The results of the analysis are shown and specifically: the maximum total reinforcement in a normalized form (K_i and K_{gl}), the maximum tensile strength for each layer (T_i and T_{gl}), the necessary length of the reinforcement (l_i and l_{gl}), the final length of the reinforcement (lf_i and lf_{gl}), (since as mentioned before the necessary length cannot be less than $0.7H_i$) and finally the angle that specifies the critical failure mechanism (Ω_i and Ω_{gl}). In addition, the necessary tensile strength and length of the reinforcement for local and global stability analysis are compared and the maximum ones are used for the final design.
- Static analysis for local and global failure mechanisms. Apparently, in this case the HPGA is zero and the user must choose an appropriate value for the Safety Factor of the construction.
- Calculation of the critical yield acceleration both for local and global failure mode, for plane failure, direct sliding and log spiral failure modes.

COMPARISON OF ANALYTICAL SOLUTION WITH CONVENTIONAL METHODS

In this section, calculations are carried out in order to demonstrate how the program works and how geometry, seismic loading and soil properties affect the necessary tensile strength and length of the reinforcement. The results are imported in a into a representative 2D limit equilibrium slope stability program (Slide) and the Safety Factors of the models are calculated in order to examine whether the reinforcement calculated by the analytical solution gives satisfactory values of SF. Moreover, the Strength Reduction Factor (SRF) and the developed stresses along the surface of the reinforcement are defined, with the help of a 2D elasto-plastic FESA program (Phase). This can only be achieved by performing FESA and not by analytical solutions.

Example of multi-step reinforced slope

In this example it is assumed that the soil is cohesionless with unit weight γ=20kN/m^3 and angle of soil friction φ=35°. The slope consists of five steps with slope angles: β_1=2:1, β_2=2:1, β_3=3:2, β_4=1:1 and β_5=1:1(vertical: horizontal).The height of each step is 10 m and the reinforcement consists of 20 equally spaced layers for each slope (d_i=0.5m).

Figure 4 shows the required tensile strength for different values of the seismic coefficient k_h for each step. As can be expected, the required tensile strength increases with increasing seismic coefficient k_h. Moreover, it can be noted that Step 2 requires reinforcement with larger tensile strength than Step 3 and almost the same tensile strength with Step 4. This is because Step 2 has steeper inclination than the other two steps. However, Step 5 requires the most tensile strength due to the influence of the weight of the upper steps.

In addition, Figure 4 illustrates the comparison of a one-step reinforced slope with a multi-step reinforced slope at different seismic coefficients. It is assumed that the one-step has average inclination α (Figure 1), associated with the geometrical characteristics of the multi-step slope through the following equation:

$$\tan(a)=\frac{(H_1+H_2+\ldots+H_n)}{\frac{H_1}{\tan(\beta_1)}+\frac{H_2}{\tan(\beta_2)}+\ldots+\frac{H_n}{\tan(\beta_n)}+\lambda_1+\lambda_2+\ldots+\lambda_{n-1}} \tag{15}$$

Specifically, for the case of the multi-step slope mentioned before (β_1=2:1, β_2=2:1, β_3=3:2, β_4=1:1 and β_5=1:1), the corresponding average inclination is 3:2 and the height of the one-step slope is H_{total}=50m. The soil mechanical characteristics are the same. As can be noted in Figure 4, the necessary tensile strength of the multi-step is significantly lower than the necessary tensile strength of the one-step slope. Moreover, as distance λ between the steps increases, the deviation of the required tensile strength is even higher.

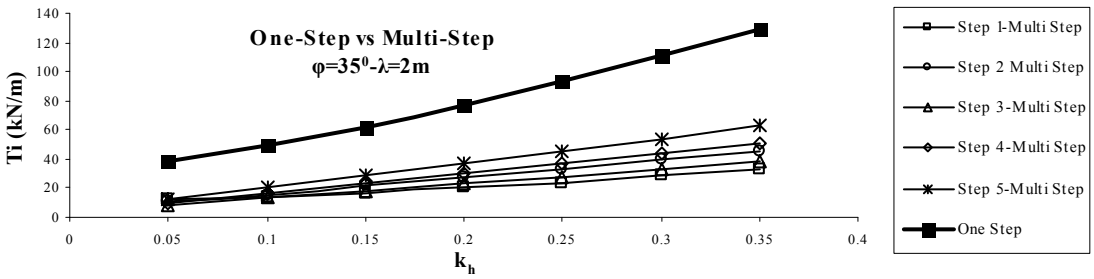

Figure 4. One-step vs. multi step, at different seismic coefficient

Limit equilibrium analysis

As mentioned before, the results of the limit analysis (required tensile strength and length of the reinforcement) are imported into a 2D limit equilibrium slope stability programme and the Safety Factors for different values of the

seismic coefficient are calculated. The calculated length of the reinforcement is different for each model (between 40-55 m), but 50 m is chosen for all of them, in order for the results to be comparable.

Models with SF higher than 1, indicate that the reinforcement is adequate against failure and models with SF lower than 1, imply that the limit equilibrium analysis is more critical that the analytical solution. In addition, due to the transient nature of ground motion, models with SF lower than 1 experience only a finite displacement rather than a complete failure. The multi-step model has the same soil geometrical and mechanical characteristics with the previous example ($H_{1\text{-}5}$=10m, γ=20kN/m^3, φ=35°, d_i=0.5m β_1=2:1, β_2=2:1, β_3=3:2, β_4=1:1, β_5=1:1). Results for different values of berm width are presented (λ=1, 2 and 3) in Table 1. The one-step model with 50 m height and average inclination α=3:2 is also analyzed (Table 1).

Specifically, Table 1 shows that for the case of the one-step slope, the reinforcement calculated by the analytical solution is adequate also for limit equilibrium analysis. For a lower seismic coefficient the SF of the multi-step models also results in values higher than 1, but as seismic coefficient increases, the SF decrease. Moreover, the required reinforcement force K is lower for multi step slopes. This proves that the construction of high slopes with steps can also be an economical and practicable solution.

It should be pointed out that the analytical solution performs limit analysis of cohesionless soils, equating the external work done with the energy dissipation (SF_{design}=1) and therefore, it is expected that SF values are close to 1. The critical failure mechanisms for SF lower than 1 are located mostly after the reinforced soil mass ends. By increasing the anchorage length of the reinforcement or by improving the mechanical characteristics of the soil between the reinforcement layers, this problem can be resolved.

Table 2, demonstrates SF of the multi step slope models that are designed with the corresponding reinforcement of the one- step slope model. As can be seen, the SF increases significantly and especially for lower values of the seismic coefficient. In the case where the construction demands larger SF, the multi step can be designed with the correspondent reinforcement of the one step, thereby increasing the SF with a lower amount of reinforcement and therefore lower cost. Table 3, shows such an example, where the reinforcement of the one-step slope (for design with smaller seismic coefficient) is adequate also for the multi step at higher seismic coefficient.

Table 1. Verification of Limit Analysis design through Bishop Analysis

Seismic Coefficient	**One Step**		**Multi Step**					
			λ=1m		**λ=2m**		**λ=3m**	
	K Limit Analysis	**F.S.** Bishop Analysis	**K** Limit Analysis	**F.S.** Bishop Analysis	**K** Limit Analysis	**F.S.** Bishop Analysis	**K** Limit analysis	**F.S.** Bishop Analysis
0.05	0.16	1.11	0.08	1.09	0.08	1.07	0.08	1.10
0.10	0.16	1.02	0.16	1.20	0.10	1.04	0.08	1.02
0.15	0.24	1.13	0.16	1.11	0.13	1.05	0.11	1.03
0.20	0.24	1.01	0.16	1.02	0.14	1.01	0.13	1.01
0.25	0.24	1.08	0.21	1.07	0.21	1.03	0.16	1.00
0.30	0.32	1.07	0.24	1.05	0.21	1.00	0.18	0.98
0.35	0.40	1.02	0.26	0.98	0.24	0.98	0.19	0.97

Table 2. Verification of Multi-Step Limit Analysis with data from One-Step Limit Analysis, through Bishop Analysis

Seismic Coefficient	**K** One Step Limit analysis	**F.S.** Multi Step - Bishop Analysis		
		λ=1m	**λ=3m**	**λ=3m**
0.05	0.16	1.22	1.26	1.30
0.10	0.16	1.20	1.21	1.21
0.15	0.24	1.27	1.26	1.28
0.20	0.24	1.20	1.18	1.20
0.25	0.24	1.13	1.10	1.15
0.30	0.32	1.09	1.08	1.10
0.35	0.40	1.05	1.04	1.05

Table 3. Bishop analysis for Multi-Step with reinforcement from One-Step at higher applied seismic coefficient

Seismic Coefficient		**F.S.** Multi Step-Bishop Analysis		
Designed Limit analysis (One Step)	**Applied** Bishop Analysis (Multi Step)	**λ=1m**	**λ=2m**	**λ=3m**
0.15	0.25	1.15	1.16	1.19
0.20	0.30	1.10	1.11	1.13
0.25	0.35	1.03	1.05	1.15

Finite Element Stress Analysis

In this section, the results of the limit analysis (required tensile strength and length of the reinforcement) are imported into a 2D elasto-plastic FESA program (Phase). The multi-step model has the same soil geometrical and mechanical characteristics as the previous example ($H_{1\text{-}5}$=10m, γ=20kN/m^3, φ=35°, d_i=0.5m, β_1=2:1, β_2=2:1, β_3=3:2, β_4=1:1, β_5=1:1). The Strength Reduction Factor (SRF) for different values of berm width is calculated (Table 4).

The basic concept of the Shear Strength Reduction method is as folows:

- The strength parameters of the slope model are reduced by a certain factor (SRF), and the finite element analysis is computed.
- This process is repeated for different values of strength reduction factor (SRF), until the model becomes unstable (the analysis results do not converge)
- This determines the critical strength reduction factor (critical SRF), or safety factor of the slope.

Table 4. Finite element stress analysis with data from limit analysis design

Seismic Coefficient	One Step		Multi Step					
			λ=1m		λ=2m		λ=3m	
	K Limit Analysis	SRF FESA	K Limit Analysis	SRF FESA	K Limit Analysis	SRF FESA	K Limit Analysis	SRF FESA
0.05	0.16	1.07	0.08	1.07	0.08	1.08	0.08	1.05
0.10	0.16	1.02	0.16	1.15	0.10	1.07	0.08	1.00
0.15	0.24	1.08	0.16	1.08	0.13	1.03	0.11	1.03
0.20	0.24	1.01	0.16	1.06	0.14	1.06	0.13	1.02
0.25	0.24	1.05	0.21	1.05	0.21	1.05	0.16	0.99
0.30	0.32	1.04	0.24	0.98	0.21	0.99	0.18	0.98

Table 4, demonstrates the SRF of the one step and multi step models. Models with SRF higher than 1, indicate that the reinforcement is adequate against failure and models with SRF lower than 1, imply that the finite element stress analysis is more critical that the analytical solution. In addition, due to the transient nature of ground motion, models with SRF lower than 1 experience only a finite displacement rather than a complete failure.

Figure 5 shows an example of finite element stress analysis. As can be noted, the potential failure surface is located after the reinforced soil mass ends and especially near the lower steps of the slope. In addition, Figures 6&7 demonstrate the allocation of the axial forces along the surface of the reinforcement layers for the same example. As can be seen, for the upper step of the slope, the axial forces have maximum values (20kN) near the middle of the reinforcement layers and then decrease. For the lower step of the slope, some of the axial forces have higher values (up to 40kN) as expected, since the influence of the soil weight on this step is much higher. However, in all cases the axial forces have lower values than the tensile strength of the reinforcement, which shows that the amount of the reinforcement calculated by the analytical solution is adequate against failure.

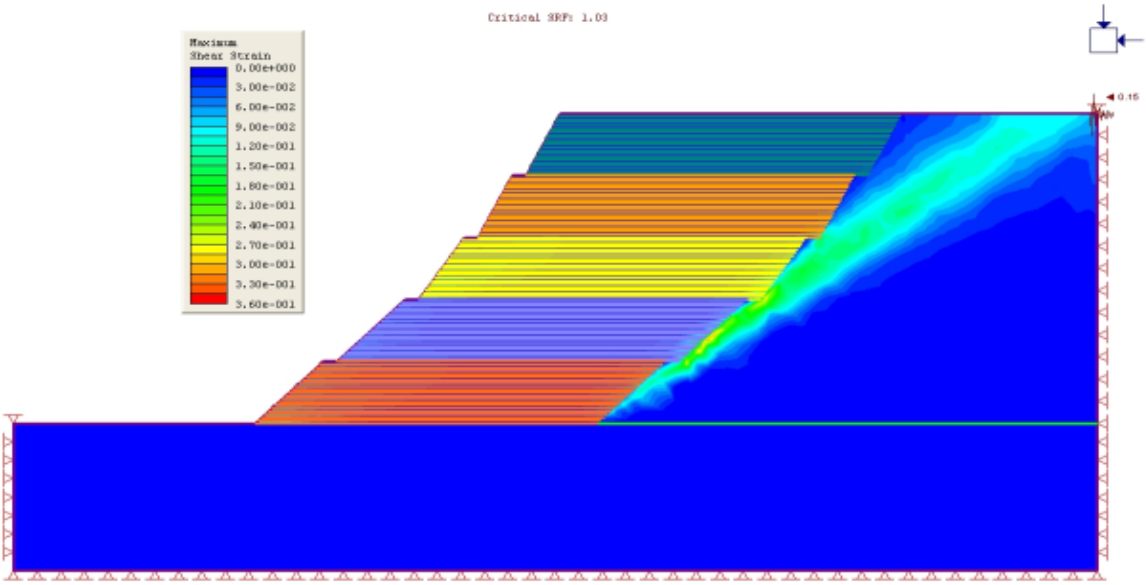

Figure 5. Finite Element Stress Analysis of multi-step reinforced slope at k_h=0.15

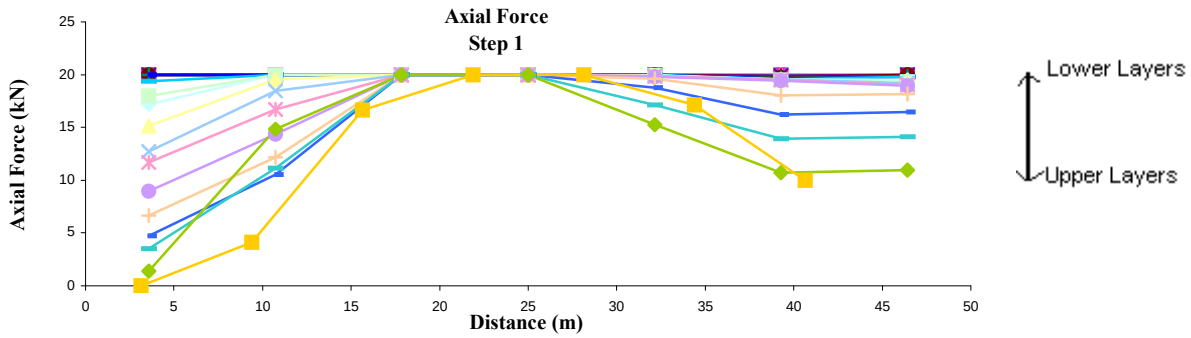

Figure 6. Allocation of Axial Force along the reinforcement layers at k_h=0.15 (Step 1- Calculation at Gauss Points)

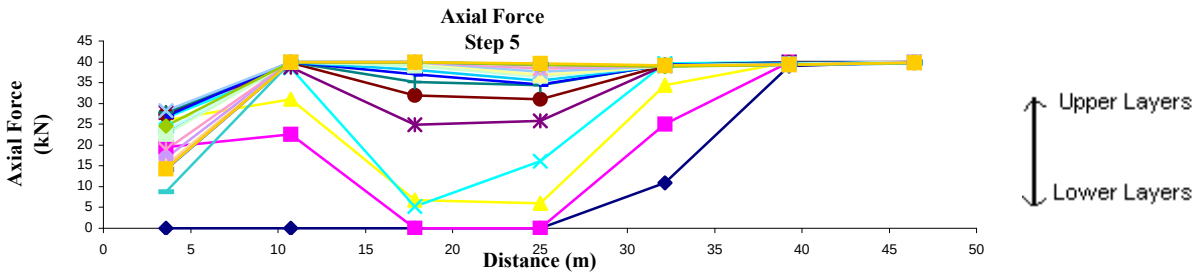

Figure 7. Allocation of Axial Force along the reinforcement layers at k_h=0.15 (Step 5- Calculation at Gauss Points)

CONCLUDING REMARKS

The expressions presented in this study based on the kinematic theorem of limit analysis can be conveniently used for the design of high reinforced slopes with steps. The tensile strength and length of the reinforcement calculated for both local and global failure modes can provide satisfactory Safety Factors when limit equilibrium analysis is performed. In addition, finite element stress analysis confirms the fact that the amount of the reinforcement calculated by the analytical solution is adequate against failure and especially at lower seismic coefficients. Finally, the necessary tensile strength of the multi-stepped slope compared to that of the one-step slope is significantly reduced, while the potential erosion due to the water flow is also limited.

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