USING SYNTHETIC DATA FROM NUMERICAL MODELLING TO VERIFY THE K-STIFFNESS METHOD FOR REINFORCED SOIL WALLS

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Abstract: The K-stiffness Method has been proposed as a new working stress method for the design of geosynthetic reinforced soil retaining walls. The method is an empirical-based approach that nevertheless retains features of current deterministic design approaches. The method has been calibrated against a database of full-scale field walls. The method has been demonstrated to give very much more accurate predictions on average of reinforcement loads under typical operational conditions than current UK and North American methods that use a Tie Back Wedge approach. However, a disadvantage of the K-stiffness Method is reliance on the use of field measurements from carefully instrumented and monitored walls of which there are only about 30 suitable structures available worldwide. A strategy to increase the database of measurements is to generate synthetic data using a verified numerical code. Carefully validated FLAC codes have been developed by the writers and checked against a series of full-scale instrumented walls constructed at the Royal Military College of Canada. The same program is used in the current study to generate reinforcement load data for a wide range of wall models varying with respect to wall height, reinforcement type and spacing. The paper shows that the data is consistent with load predictions using the current K-stiffness Method. The value of the paper is that it gives confidence that the numerical approach adopted by the writers can be used to extend the physical database that was originally used to develop the K-stiffness Method to a wider range of reinforced wall structures.

Keywords: geosynthetic, geogrid reinforcement, numerical, reinforced soil wall, limit state design, design method

INTRODUCTION

Current design methods to estimate reinforcement loads in geosynthetic reinforced soil walls are based on limit-equilibrium methods such as the Coherent Gravity Method in the UK (BS8006 1995) and the Simplified Method in the USA (AASHTO 2002). Both methods give similar predictions for unfactored reinforcement loads for simple wall geometries and boundary conditions. Back-analysis of measurements from instrumented full-scale laboratory and field walls has shown that the Simplified Method is very conservative with respect to the magnitude of predicted loads under typical operational conditions (e.g. Allen et al. 2003, Bathurst et al. 2005, 2008). The coefficient of variation (COV) of the ratio (bias) of measured to predicted load is so large that, from a statistical point of view, there is no relationship between observed and predicted load values. This observation has led to development of a new working stress method called the K-stiffness Method for the calculation of reinforcement loads. The method is empirical in nature with coefficients that have been back-fitted to the same database mentioned above. Bias statistics have shown that the K-stiffness Method gives better estimates of reinforcement loads (on average) and the COV on bias values is low enough to give confidence that the method is sufficiently accurate for design (Bathurst et al. 2008). Nevertheless, the amount of high quality field data available to carry out calibration of the K-stiffness method is limited. For example, the latest version of the method relies on approximately 30 case studies. A strategy to increase the database of measurements is to generate synthetic data using a verified numerical code.

In this paper a FLAC code is used which has been verified against the measured results of a series of full-scale instrumented walls constructed at the Royal Military College of Canada (RMC). The code is used to generate quantitative deformation, strain and load data for a series of vertical modular block reinforced soil walls with a range of heights, reinforcement spacing and two different reinforcement materials. Bias statistics for numerically computed loads and predicted values are shown to be similar to bias statistics using measured values from the physical wall database of vertical walls and predictions using the Coherent Gravity Method and the K-stiffness Method. This gives confidence that synthetic data from numerical simulations can be used to generate data to fill in the gaps in the physical database that is currently available to investigate the accuracy of current design methods and to calibrate the K-stiffness Method.

NUMERICAL MODEL

General

Numerical models for the RMC physical walls using the program FLAC (Itasca 2005) have been reported by Hatami and Bathurst (2005, 2006). A modified version of these earlier codes is used in the current study. The accuracy of the current code was first checked by confirming that it gave satisfactory quantitative agreement with measurements taken from two of the RMC test walls.

Details of the physical RMC test walls are reported by Bathurst et al. (2000, 2006). The hard-faced walls in the RMC test program used a dry-stacked column of solid masonry concrete blocks together with a sand backfill. The soil reinforcement products used in the walls were integral drawn polypropylene (PP) geogrid, woven polyester (PET) geogrid and welded wire mesh.

The bottom-up construction process in the physical RMC model tests was modelled using sequential 0.15-m thick layers. The moving local datum in the physical tests was captured as each row of facing units and soil layer was placed over the previous lift in the numerical model. The influence of soil compaction was simulated by applying a transient uniform pressure to each soil lift as described by Hatami and Bathurst (2005, 2006).

The current paper presents simulation results for vertical face, modular block walls with height of H = 3.6, 6.0 and 9.0 m, reinforcement spacing of S_v = 0.3, 0.6 and 0.9 m and two different reinforcement products. Figure 1 shows an example FLAC numerical grid used in the numerical simulations.

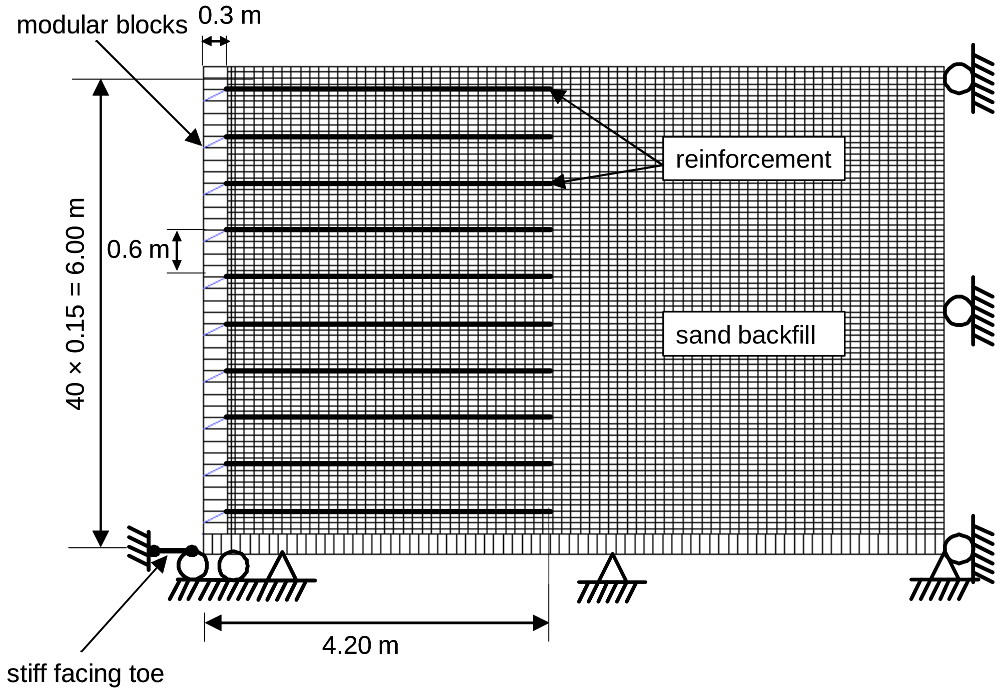

Figure 1. Example FLAC numerical grid for 9 m-high wall

Material properties

The soil reinforcement layers in this study were simulated using FLAC cable elements. The properties of one material were selected to match a uniaxial high-density polyethylene (HDPE) geogrid and the other a woven PET geogrid. The stiffness of the HDPE geogrid is strain level and time dependent whereas the stiffness of the PET geogrid is essentially constant at the strain levels of interest in the current study. The load-strain-time properties of both materials under constant or monotonically increasing load can be expressed by the hyperbolic function proposed by Hatami and Bathurst (2006):

$$T(\varepsilon,t)=\left(\frac{1}{\dfrac{1}{J_0(t)}+\dfrac{\eta(t)}{T_f(t)}\varepsilon}\right)\varepsilon$$

Where: $J_o(t)$ is the initial tangent stiffness, $\eta(t)$ is a scaling function, $T_f(t)$ is the stress-rupture function for the reinforcement and, t is time (i.e. duration of loading). Parameter values are presented in Table 1. The secant stiffness ($J_s(\varepsilon,t) = T(\varepsilon,t)/\varepsilon$) of the HDPE product is about 2.5 times as large as the corresponding value for the PET product.

The soil is modelled as a non-linear elastic material with a hyperbolic stress-strain function. The stress-dependent elastic tangent modulus is expressed as (Duncan et al. 1980):

$$E_t=\left[1-\frac{R_f(1-\sin\varphi)(\sigma_1-\sigma_3)}{2c\cos\varphi+2\sigma_3\sin\varphi}\right]^2 K_e p_a\left(\frac{\sigma_3}{p_a}\right)^n$$

Where: σ_1 = major principle stress, σ_3 = minor principle stress, p_a = atmospheric pressure and the other parameters are defined in Table 2. These parameters have been selected to match the SW sand material reported by Boscardin et al. (1990) with some adjustments.

Hatami and Bathurst (2005) showed that the Duncan-Chang parameters back-fitted from triaxial tests on the RMC sand under-estimated the stiffness and strength of the same soil when tested in a plane strain test apparatus. To overcome this deficiency, while retaining a simple non-linear elastic constitutive model for the soil, the bulk modulus formulation proposed by Boscardin et al. (1990) is used in the current study:

$$B_t = B_i\left[1+\frac{\sigma_m}{B_i\varepsilon_u}\right]^2$$

Here: σ_m = mean pressure = $(\sigma_1+\sigma_2+\sigma_3)/3$; B_i and ε_u are material properties that are determined as the intercept and the inverse of slope from a plot of $\sigma_m/\varepsilon_{vol}$ versus σ_m, respectively, in an isotropic compression test, and ε_{vol} is volumetric strain. In the user-defined model in this study, the B_t value was restricted to the following range:

$$\frac{E_t}{\left(1-2\nu_{tmin}\right)} \le B_t \le \frac{E_t}{\left(1-2\nu_{tmax}\right)}$$

Where Poisson's ratio can vary between the limits $\nu_{t\,max} = 0.49$ and $\nu_{t\,min} = 0$.

Table 1. Reinforcement properties

Reinforcement type	Hyperbolic stiffness function parameters for t = 1000 h			Ultimate (index) strength*
	J_o (t) (kN/m)	η(t)	T_f (t) (kN/m)	T_y (kN/m)
PET	285	0	0	80
HDPE	800	0.7	35	72

*Based on peak strength measured during 10% strain/minute constant-rate-of-strain (CRS) test.

Table 2. Backfill soil properties

Model parameters	Value
K_e (elastic modulus number)	950
K_{ur} (unloading-reloading modulus number)	1140
B_i/P_a (initial bulk modulus)	74.8
ε_u (asymptotic volumetric strain value)	0.02
n (elastic modulus exponent)	0.7
R_f (failure ratio)	0.8
ν_t (tangent Poisson's ratio)	0-0.49
ϕ (friction angle) (degrees)	48
c (cohesion) (kPa)	0.2

Interface and boundary conditions

The interfaces at the facing column-backfill, block-block, foundation-backfill, and reinforcement-backfill were modelled as linear spring-slider systems with interface shear strength defined by the Mohr-Coulomb failure criterion. The interface shear stiffness between modular blocks was assumed as 40 to 80 MN/m/m based on results of direct shear tests carried out on the solid masonry blocks and varied with height of wall face above the interface.

The reinforcement (cable) elements were assumed to be bonded to the backfill using the FLAC grout utility. The large bond strength along the reinforcement-backfill interface was selected to prevent slip and to simplify the model. Measured reinforcement displacements showed that this is a reasonable assumption under working load conditions for the combination of reinforcement products and compacted sand used in the RMC physical tests.

A fixed boundary condition in the horizontal direction was assumed at the numerical grid points on the backfill far-end boundary, matching the back of the RMC test facility. A fixed boundary condition in both horizontal and vertical directions was used at the foundation level matching the test facility concrete strong floor. The toe of the facing column was restrained horizontally by a very stiff spring element with K_{toe} = 4 to 20 MN/m/m matching the measurement at this boundary in the RMC physical tests. This value was varied in the simulations depending on the height of wall and reinforcement type. Interface properties are summarized in Table 3.

Table 3. Interface properties (modified from Hatami and Bathurst 2005, 2006)

Interface	Value
Soil-Block	
δ_{sb} (friction angle) (degrees)	48
ψ_{sb} (dilation angle) (degrees)	6
K_{nsb} (normal stiffness) (MN/m/m)	100
K_{ssb} (shear stiffness) (MN/m/m)	1
Block-Block	
δ_{bb} (friction angle) (degrees)	57
c_{bb} (cohesion) (kPa)	46
K_{nbb} (normal stiffness) (MN/m/m)	1000
K_{sbb} (shear stiffness) (MN/m/m)	40/60/80
Backfill-Reinforcement	
ϕ_b (friction angle) (degrees)	48
s_b (adhesive strength) (kPa)	1000
K_b (shear stiffness) (kN/m/m)	1000

EXAMPLE SIMULATION RESULTS

The FLAC results from each simulation run provide detailed quantitative output of wall toe forces, foundation pressures, facing displacements, connection loads and reinforcement strains.

Figure 2 shows an example of strain distributions in selected layers for a 6.0 m-high wall. It can be seen that for the hard-faced wall type used in this study, strains in each layer are highest close to the facing of the wall and decrease to zero at the free end. For design purposes a distinction between connection strains (loads) and internal strains (loads) should be made. A general observation from strain distributions in the RMC walls and instrumented hard-faced field walls is that strains at the connections propagate along the reinforcement for a distance of about 0.8 m. This distance is called the connection zone. Beyond this distance strains and loads in the reinforcement are assumed to be developed by reinforcement-soil interaction in the reinforced soil zone. For the bottommost layers close to the heel of the facing column, the connection zone is assumed to become narrower. For analysis purposes in this paper, this zone is assumed to follow the Coulomb failure surface propagating from the heel of the facing column until a distance of 0.8 m and then to remain constant at 0.8 m over the remaining height of the wall.

Figure 3 shows internal strains plotted against normalized height for walls with different reinforcement spacing. It can be seen that a reduction in the reinforcement spacing leads to a significant reduction in internal reinforcement strains. As expected, the higher stiffness HDPE product results in lower internal strains than the PET material. When all other parameters are the same, reinforcement strains at the normalized elevation increase with height of wall. While not shown here, wall deformation profiles gave similar shapes and in the same sequence as the plots in Figure 3. The grey shaded symbols in Figure 3 correspond to walls that exhibited excessive wall deformations taken as > 30 mm over the height of the wall or reinforcement strains greater than 3%. These criteria follow recommendations by Allen et al. (2003) who noted that when these threshold values were exceeded for reinforced soil walls with granular backfill, poor wall performance typically ensued. Later in the paper we refer to these limits as serviceability criteria.

COMPARISON OF SYNTHETIC DATA WITH CURRENT ANALYTICAL DESIGN MODELS

Unfactored reinforcement loads can be estimated using the Tie Back Wedge Method as described in BS8006 (1995). In this method the Meyerhof approach is used to compute vertical stresses at the reinforcement elevation. A tributary area approach is then used to assign active earth pressures over each layer spacing (S_v). The integrated pressure over S_v is then assumed equal to the maximum tensile load (T_{max}) in the reinforcement layer. The method is the same as the AASHTO (2002) Simplified Method if the Meyerhof calculation is omitted.

The computation of the maximum internal reinforcement load using the empirical-based K-stiffness Method in its most current presentation is expressed as (Bathurst et al. 2008):

$$T_{max} = \frac{1}{2} K\gamma \ (H + S)\ S_v \ D_{tmax} \Phi_g \ \Phi_{local} \ \Phi_{fs} \ \Phi_{fb} \ \Phi_c$$

Here: K = lateral earth pressure coefficient; γ = unit weight of the soil; H = height of the wall; S = equivalent height of uniform surcharge pressure q (i.e., $S = q/\gamma$); S_v = tributary area (equivalent to the vertical spacing of the reinforcement

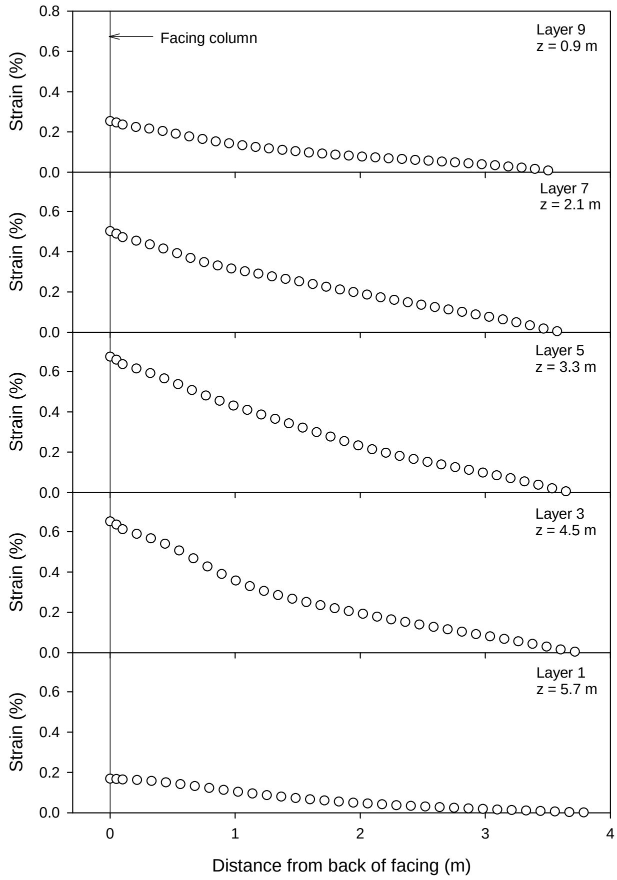

Figure 2. Strain distribution for selected layers (HDPE, H = 6.0 m, S_v = 0.6 m)

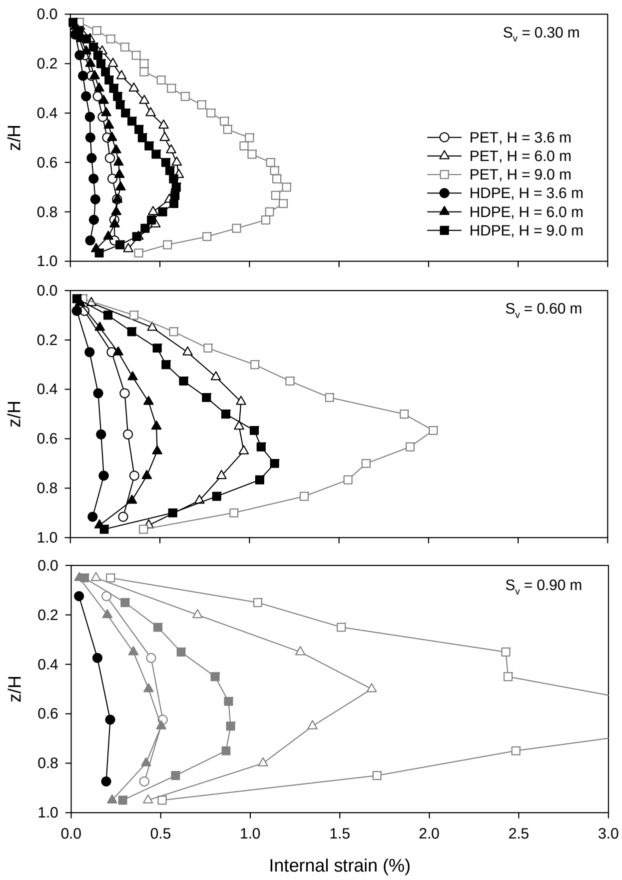

Figure 3. Calculated reinforcement strains at end of construction. Note: grey symbols correspond to walls with excessive deformation (> 30 mm) or strain (> 3%)

in the vicinity of each layer when analyses are carried out per unit length of wall); D_{tmax} = load distribution factor that modifies the reinforcement load based on layer location. The remaining terms, Φ_g, Φ_{local}, Φ_{fs} and Φ_{fb} are influence factors that account for the effects of global and local reinforcement stiffness, facing stiffness and face batter, respectively. The coefficient of lateral earth pressure is calculated as K = 1 - sinϕ with $\phi = \phi_{ps}$ = secant peak plane strain friction angle of the soil. However, it should be noted that parameter K is used as an index value and does not imply that at-rest soil conditions exist in the reinforced soil backfill according to classical earth pressure theory.

Measured reinforcement loads from instrumented vertical hard-faced walls in the most current database available to the authors (Bathurst et al. 2008), have been computed as the product of measured strain and a suitably selected secant stiffness value that is a function of duration of loading and strain. This dataset comprises a subset of 11 walls that have in common that they are vertical hard-faced walls constructed with polymeric reinforcement and granular (frictional) backfill soils. This subset of the larger database available to the writers is attractive because most reinforced soil walls are vertical or near vertical and are constructed with a hard facing. The numerical simulations comprise a much narrower range of properties and geometries but fall within the same category of field structures.

For the synthetic database in the current study, the internal reinforcement load (T_{max}) is computed as the product of the secant stiffness function J(t,ε) of the material with time taken as 1000 h, and the maximum strain computed at a location behind the connection zone. In this paper we adopt the same nomenclature for the maximum reinforcement load from all reinforcement layers as in earlier related papers (i.e. T_{mxmx}). The number of T_{max} data points for hard-faced vertical walls is 222 and for T_{mxmx} is 18. The value of T_{mxmx} is of practical interest since it is common practice to compute the maximum reinforcement load in a wall and then use this value to select a single reinforcement product for all layers to simplify the design.

Figures 4a and 4b show plots of predicted versus measured or calculated values of T_{mxmx} . Predicted loads are unfactored reinforcement loads computed using the Tie Back Wedge Method and the K-stiffness Method. Here, measured values refer to values computed from strains measured in monitored structures (for cases from the physical database). Computed values refer to values from the numerical simulations. The data plots have been produced with logarithmic axes to reduce visual clutter at low load levels and because the measured and predicted loads vary by at least one order of magnitude. It is important to recall that the physical data points correspond to monitored vertical-faced field walls with reinforcement properties, layer spacing and soil properties that are different from those used in the numerical simulations. However, from a practical point of view, both sets of data fall within a similar category of structure (i.e. non-surcharged, vertical hard-faced walls constructed with polymeric reinforcement materials and a granular (frictional) backfill). Both plots show that the numerical data points and physical data points generally overlap. In Figure 4a the measured and calculated values of T_{mxmx} are visually much less than the predicted values using the Tie Back Wedge Method. Qualitatively, both measured (physical) and numerical results show a similar level of conservatism (for design) using the Tie Back Wedge Method. This is interpreted as evidence that the model employed in the numerical simulation work gives reasonable predictions when these predictions are compared to predictions using current closed-form (analytical) solutions (i.e. Tie Back Wedge Methods).

A similar comparison is shown in Figure 4b where physical and numerical reinforcement loads are compared to predicted values using the K-stiffness Method. In this case there is better visual overlap between numerical and physical data points and both sets of data fall within a much narrower band that is close to the one-to-one correspondence line. These data show that the K-stiffness Method is more accurate than the previous method when both physical and numerical data are compared to predicted values.

While there is qualitative visual evidence that numerical simulations give similar predictions to analytical solutions for the same category of structure, the correspondence between numerical and physical test data can be quantified using statistics for the load bias values (i.e. ratio of measured to predicted loads). Here "measured" means loads deduced from strain measurements taken from field walls or calculated loads in the case of synthetic data. Bias statistics are summarized in Table 4 for all data and for selected subsets of the synthetic (numerical) datasets. The bias statistics for mean and COV values for the physical datasets show that the Tie Back wedge Method is not accurate. For example, measured loads are, on average, one third of the predicted values and the spread in bias values quantified by COV is in the range of 43 to 53%. The corresponding statistics for the K-stiffness Method are 0.82 to 1 for the mean bias value and a range of 21 to 37% for the COV of T_{mxmx} and T_{max} bias values, respectively. For design, a mean bias value close to one is desirable together with a small COV value. When bias statistics using synthetic data and the Tie Back Wedge Method are computed, slightly better mean and COV bias values result. Nevertheless, bias values on average are very conservative (i.e. analytical values at least a factor of two lower than numerical predicted values). Bias statistics using synthetic data and the K-stiffness Method are much better. The mean bias values are 1.08 and 0.95 for T_{mxmx} and T_{max} and the corresponding COV values are 12 and 19%. The generally lower COV values for bias values using synthetic data compared to physical data is the result of a much narrower range of input parameters compared to the set of physical data used in the comparison. For example, in the numerical dataset there is only one backfill type while there is a range of soil backfills in the 11 case studies that comprise the physical dataset.

There are other observations that can be made from the bias statistics in Table 4. There is decreasing number of T_{mxmx} data points as the layer spacing increases. This is because the number of instances of excessively high wall deformations or strain according to the serviceability criteria introduced earlier are greater. Another observation is that the bias statistics for synthetic data and the Tie Back Wedge Method are in all but one case slightly better than the bias statistics using physical data. The writers observed during examination of reinforcement load distribution with depth below the wall crest that walls with smaller spacing tended to better approach a linear distribution with depth. This general trend is predicted by the Tie Back Wedge Method. The physical database against which the K-stiffness Method was calibrated was largely based on walls with larger spacing (i.e. 0.6 to 1 m). Comparison of the statistics for HDPE and PET simulations shows that the bias statistics for numerical results and Tie Back Wedge Method predictions are better for the HDPE material than the less stiff PET material. This may be expected when it is noted that the Tie Back Wedge Method (at least in North America) was developed initially for very stiff reinforcing elements (e.g. steel strips) (Bathurst et al. 2007).

CONCLUSIONS

The validity of current design methods for estimation of reinforcement loads under operational conditions is best undertaken by comparing predictions to measured values from carefully instrumented and monitored full-scale field or laboratory structures. However, there is limited number of physical case studies available to establish the quantitative accuracy of any current or proposed design method for this purpose. An obvious approach to generate additional data to fill in the gaps in the physical database is to use the results of numerical simulations. However, before confidence in a set of numerical codes can be developed, they must first be tested and verified by comparison with individual comprehensively instrumented and monitored full-scale tests (e.g. Hatami and Bathurst 2005, 2006). A second step, which is the focus of this paper, is to check that the numerical simulation generate load values that are consistent from a statistical point view with the results of physical datasets.

In this paper a numerical FLAC code previously verified by Hatami and Bathurst (2005, 2006) is shown to generate bias statistics for measured to predicted loads that are similar to values computed using two different analytical models and physical measurements. The two analytical methods are the Tie Back Wedge Method and the K-stiffness Method. This is a novel strategy which recognizes that a numerical simulation technique is valuable if it can produce data that has similar statistical characteristics to data corresponding to the same class of structures in a limited physical dataset. This is particularly important when the objective is generate bias statistics which can be used for limit states design calibration (Allen et al. 2005).

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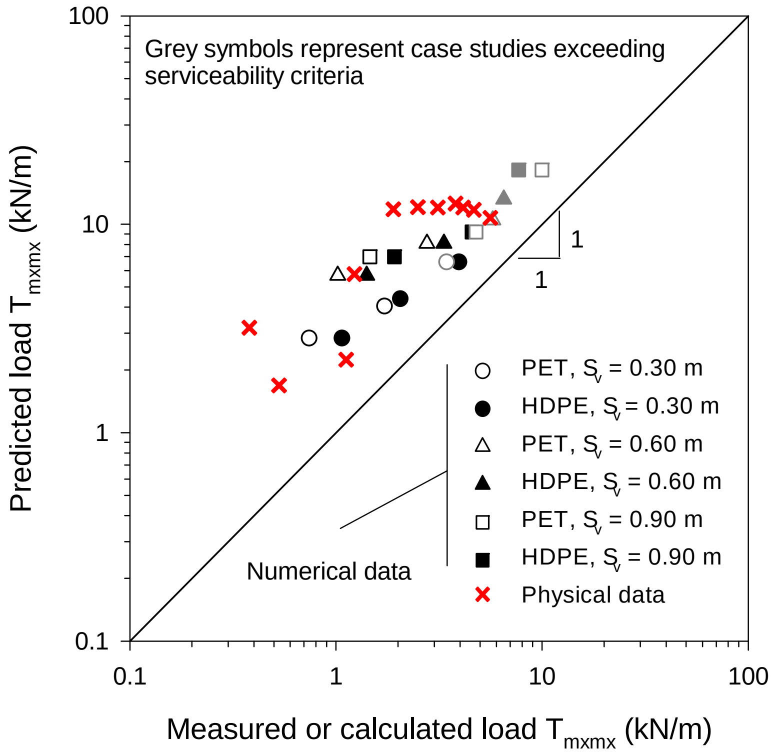
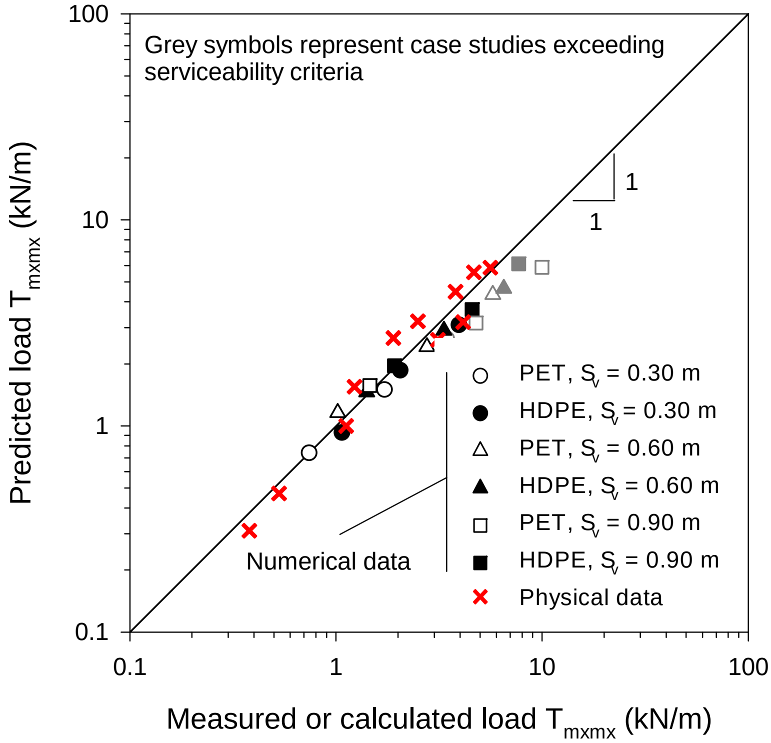


a) Tie Back Wedge Method



b) K-stiffness Method

Figure 4. Predicted versus measured or calculated maximum internal reinforcement loads

Defence (Canada) and the following US State Departments of Transportation: Alaska, Arizona, California, Colorado, Idaho, Minnesota, New York, North Dakota, Oregon, Utah, Washington and Wyoming.

Table 4. Summary of statistics for ratio (bias) of measured to predicted reinforcement loads

	Wall cases	Load	Number of data points*	Tie Back Wedge Method		K-stiffness Method	
				Bias statistics			
		(kN/m)		Mean	COV (%)	Mean	COV (%)
Physical data	All walls	T_{mxmx}	11	0.30	43	1.00	21
		T_{max}	49	0.29	53	0.82	37
Numerical results	All walls	T_{mxmx}	12	0.36	36	1.08	12
		T_{max}	136	0.46	41	0.95	19
	PET only	T_{mxmx}	5	0.28	36	1.02	12
		T_{max}	50	0.37	41	0.95	16
	HDPE only	T_{mxmx}	7	0.41	30	1.12	11
		T_{max}	86	0.51	37	0.94	20
	S_v = 0.3 m	T_{mxmx}	5	0.43	29	1.14	9
		T_{max}	89	0.50	35	0.93	19
	S_v = 0.6 m	T_{mxmx}	4	0.29	35	1.03	13
		T_{max}	32	0.39	48	0.97	17
	S_v = 0.9 m	T_{mxmx}	3	0.33	46	1.05	16
		T_{max}	15	0.37	57	1.00	22

* Vertical walls satisfying serviceability criteria

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