THE GENERAL THEORY OF THERMOELASTIC MULTILAYERS ROAD PLATES, REINFORCED WITH GEOSYNTHETIC MATERIALS

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ABSTRACT: The theory of multilayered constructive - anisotropic plates on the elastic basis is stated. Each layer of a plate has individual structure of reinforcing as cellular confined systems, geogrids, geonets or geo-textiles.. The elastic basis resists both normal, and shear deformations. The general system of equations of a longitudinal - cross bending of a multilayered plate of asymmetrical structure on height is constructed.
On the basis of the developed models of mechanical deformation of the elastic layer reinforced with cellu-lar confined systems, geogrids, geonets, the mathematical device is received, allowing to estimate effect from application various structures at reinforcing multilayered designs.
The problem of a longitudinal - cross bending of the polyreinforced plate is shown to a problem of a cross bending and solved by Krylov's method for a case of a cylindrical bend. The numerical decision qualitatively reflecting effect from reinforcing of road pavements by cellular confined systems is received

1 INTRODUCTION

Reinforcing of soils with geosynthetic materials takes place in practice of road construction more widely as autors of proceedings of EuroGeo1 (1996), EuroGeo 2 (2000), proceedings of Sojuz-dornii (1998), (2001), L'VOVICH (1998) have alrea-dy mentioned.

Methods of calculation of pavements and the reinforced bases are developed basically by empirical way. Common fault of existing methods is traditional design procedures with continuous, homogeneous, isotropic layers attraction for calculation of the reinforced designs being in essen-ce constructive - anisotropic.

Strictly speaking, soils, synthetic materials of which reinforcing structures are made, have no properties of ideally elastic material, and their diagram of deformation – not linear. Therefore the stated approach to construction of settlement model of the reinforced ground in elastic statement is considered by us as the first approximation which further will be improved.

2 GEOMETRICAL, PHYSICAL AND STATIC EQUOTIONS

Let's consider the multilayered plate consisting from m of layers. We shall designate thickness of layers h1, h2, ., hm. Let coordinate plane XY coincides with the top surface of a package of layers, axis Z is directed downwards (fig. 1) and the next equations takes place

$$H_m = \sum_{i=1}^{m} h_i,\ \ H_{i-1} = h_1 + h_2 + \ldots + h_{i-1},\ \ H_i = H_{i-1} + h_i. \qquad (1)$$

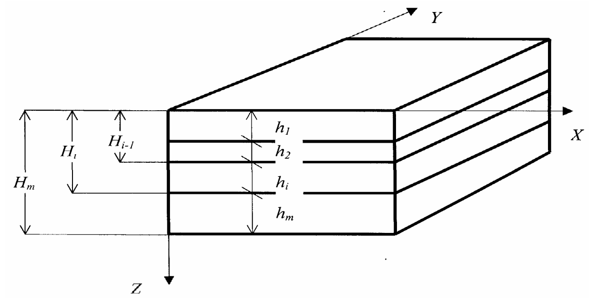

Fig. 1. The scheme of an arrangement of layers

Let's assume, that for all package of the elastic layers rigidly connected among themselves, hypotheses of Kerhoff-Love are fair. Thus physical parities should be carried out separately for each layer by virtue of that different layers can have various structure of reinforcing and consequently, various characteristics thermoelasticity:

$$\bar{u} = C\bar{u}_0;\ \ \bar{\varepsilon} = D\bar{u} = DC\bar{u}_0;$$
$$\bar{\sigma}^i = A^i\bar{\varepsilon} - A^i_T T;\ \ A^i_T = A^i\bar{\alpha}^i. \qquad (2)$$

Here $\bar{u}$ - a vector of movings of an any point of a plate; $\bar{u}_0$ - a vector of movings of points of coordinate plane XY; $\bar{\varepsilon}$ - a vector of deformations; $\bar{\sigma}^i, \bar{\alpha}^i$ - vectors of strenghs and coefficients of temperature deformation in "i" layer; A^i_T - a matrix of thermoelastic characteristics:

$$\bar{u}=(u,v,w)^T;\ \ \bar{u}_o=(u_o,v_o,w)^T;\ \ \bar{\varepsilon}=(\varepsilon_x,\varepsilon_y,\gamma_{xy})^T;$$

$$\bar{\sigma}^i=(\sigma^i{}_x,\sigma^i{}_y,\tau^i{}_{xy})^T;\ \ \bar{\alpha}^i=(\alpha^i_x,\alpha^i_y,\alpha^i_{xy})^T; \qquad (3)$$

$$A^i_T=\left[A^i_{1T},A^i_{2T},A^i_{3T}\right]^T.$$

Here C,D – are the matrixes of differentiation; A^i – the matrix of elastic characteristics in "i" layer:

$$C=\begin{bmatrix}1 & 0 & -z\,\partial/\partial x\\ 0 & 1 & -z\,\partial/\partial y\\ 0 & 0 & 1\end{bmatrix};\ \ D=\begin{bmatrix}\partial/\partial x & 0 & 0\\ 0 & \partial/\partial y & 0\\ \partial/\partial y & \partial/\partial x & 0\end{bmatrix};$$

$$A^i=\begin{bmatrix}A^i{}_{11} & A^i{}_{12} & A^i{}_{13}\\ A^i{}_{21} & A^i{}_{22} & A^i{}_{23}\\ A^i{}_{31} & A^i{}_{32} & A^i{}_{33}\end{bmatrix}. \qquad (4)$$

Expressions (2) - (4) are written down for the general case, when a material of "i" layer is anisotropic. In ortotropic material factors A^i_{13} , A^i_{23} , A^i_{3T} , α^i_{xy} are absent.

On thickness of "i" layer the temperature changes according to linear dependence

$$T^i(z)=t_{i0}+t_{i1}z\,, \qquad (5)$$

Where t_{i0} , t_{i1} - is the given constants.

The differential equations of balance of an infinite-small element of a plate are

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}+q^-_x-q^+_x+X=0;$$
$$\frac{\partial N_{yx}}{\partial x}+\frac{\partial N_y}{\partial y}+q^-_y-q^+_y+Y=0; \qquad (6)$$
$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}+q^-_z-q^+_z+Z=0.$$

Here N_x,N_y,N_{xy} – are tensile and shearing forces in a plate:

$$N_x=\sum_{i=1}^{m}\int_{H_{i-1}}^{H_i}\sigma^i{}_x dz;\ \ N_y=\sum_{i=1}^{m}\int_{H_{i-1}}^{H_i}\sigma^i{}_y dz;\ \ N_{xy}=\sum_{i=1}^{m}\int_{H_{i-1}}^{H_i}\tau^i{}_{xy}dz, \qquad (7)$$

Q_x , Q_y - cross forces in a plate:

$$Q_x=\sum_{i=1}^{m}\int_{H_{i-1}}^{H_i}\tau^i{}_{zx}dz;\ \ Q_y=\sum_{i=1}^{m}\int_{H_{i-1}}^{H_i}\tau^i{}_{zy}dz, \qquad (8)$$

X , Y, Z – projections to coordinate axes equally ef-fective the volumetric forces, taken on thickness of a plate;

q^-_x,q^-_y,q^-_z - external loading on a plate (at z = 0);

q^+_x,q^+_y,q^+_z - reaction of the elastic basis (at z = Hm):

$$q^+_z=-C_z w,\ \ q^+_x=C_x+f_x C_z w,$$
$$q^+_y=C_y+f_y C_z w. \qquad (9)$$

Here Cz – is the factor of bed, Cx, Cy - factors of coupling; fx, fy - factors of friction.

If the "i" layer of small thickness $h_i=\delta_i$ is reinforced with geonet, or geogrid or geotextile, we propose, that stresses $\sigma^i{}_x$ $\sigma^i{}_y$ and $\tau^i{}_{xy}$ on thickness of the given layer are constants, therefore in the equations (7),), in components, concerning to the "i" layer, we shall make replacement of integrals.

$\int_{H_{i-1}}^{H_i}\sigma^i_x dz,\ \int_{H_{i-1}}^{H_i}\sigma^i_y dz$ и $\int_{H_{i-1}}^{H_i}\tau^i_{xy}dz$ with products accordingly $\sigma^i_x\delta_i,\ \sigma^i_y\delta_i$ и $\tau_{xy}{}^i\delta_i$.

First two equations (6) we shall multiply on zdz, integrate on z and after transformations in view of equations (9) we shall receive following:

$$\frac{\partial M_x}{\partial x}+\frac{\partial M_{xy}}{\partial y}-H_m q^+_x+R_x=Q_x;$$
$$\frac{\partial M_{yx}}{\partial x}+\frac{\partial M_y}{\partial y}-H_m q^+_y+R_y=Q_y. \qquad (10)$$

Here M_x,M_y,M_{xy} – are the bending and twisting moments in the multilayered plate, determined under formulas

$$M_x=\sum_{i=1}^{m}\int_{H_{i-1}}^{H_i}\sigma^i_x zdz;\ M_y=\sum_{i=1}^{m}\int_{H_{i-1}}^{H_i}\sigma^i_y zdz;$$
$$M_{xy}=\sum_{i=1}^{m}\int_{H_{i-1}}^{H_i}\tau^i_{xy}zdz, \qquad (11)$$

R_x,R_y – the total moments, created by volumetric forces.

If the "j" layer of small thickness $h_j=\delta_j$ is reinforced with geonet, or geogrid or geotextile, we propose, that stresses $\sigma^j x$ and $\sigma^j y$ on thickness of the given layer are constants. therefore in the first and second equations (11), in components, concerning to the "j" layer, we shall make replacement of

integrals $\int_{H_{j-1}}^{H_j}\sigma^j_x zdz$ and $\int_{H_{j-1}}^{H_j}\sigma^j_y zdz$ with products

accordingly $\sigma^j_x\delta_j H_j$ и $\sigma^j_y\delta_j H_j$.

If we substitute equations (7) in (6), and also take in account (2) and (5), after integrating we shell obtain

$$\bar{N}=\bar{N}_o-\bar{N}_T;\ \ \bar{N}=(N_x,N_y,N_{xy})^T;$$
$$\bar{N}_o=(N_{xo},N_{yo},N_{xyo})^T;\ \ \bar{N}_T=(N_{xT},N_{yT},N_{xyT})^T \qquad (12)$$

where $\overline{N}$ - is the vector of tensile and shearing stresses; $\overline{N}_o$ - vector of the same stresses, caused by tensile and bending strains; $\overline{N}_T$ - the same, from temperature influences.

Components of a vector $\overline{N}_o$:

$$N_{so} = b_{k1}\varepsilon_{ox} + b_{k2}\varepsilon_{oy} + b_{k3}\gamma_{oxy} + c_{k1}\chi_x + c_{k2}\chi_y + c_{k3}\chi_{xy}, \ (s = x, y, xy; k = 1,2,3). \quad (13)$$

Components of a vector $\overline{N}_T$:

$$N_{sT} = \sum_{i=1}^{m} A^i_{kT} t_{i0} h_i + \sum_{i=1}^{m} A^i_{kT} t_{i1} p_i, \quad (s = x, y, xy; k = 1,2,3). \quad (14)$$

Constants p_i in formulas (14) are determined from expressions (19).

Substituting (11) in (10), and also taking into account dependences (2) and (5), after integration we shall receive

$$\overline{M} = \overline{M}_o - \overline{M}_T; \ \overline{M} = (M_x, M_y, M_{xy})^T;$$
$$\overline{M}_o = (M_{xo}, M_{yo}, M_{xyo})^T; \overline{M}_T = (M_{xT}, M_{yT}, M_{xyT})^T. \quad (15)$$

Here $\overline{M}$ - is the vector of bending and twisting moments; $\overline{M}_o$ - vector of the same stresses, caused by tensile and bending strains; $\overline{M}_T$ - the same, from temperature influences.

Components of a vector $\overline{M}_o$:

$$M_{so} = c_{k1}\varepsilon_{ox} + c_{k2}\varepsilon_{oy} + c_{k3}\gamma_{oxy} + d_{k1}\chi_x + d_{k2}\chi_y + d_{k3}\chi_{xy}, \ (s = x, y, xy; k = 1,2,3). \quad (16)$$

Components of a vector $\overline{M}_T$:

$$M_{sT} = \sum_{i=1}^{m} A^i_{kT} t_{i0} p_i + \sum_{i=1}^{m} A^i_{kT} t_{i1} g_i, \quad (s = x, y, xy; k = 1,2,3). \quad (17)$$

Constants $b_{kl}, c_{kl}, d_{kl} (k, l = 1,2,3)$ are determined from expressions

$$b_{kl} = \sum_{i=1}^{m} A^i_{kl} h_i, \ c_{kl} = \sum_{i=1}^{m} A^i_{kl} p_i, \ d_{kl} = \sum_{i=1}^{m} A^i_{kl} g_i. \quad (18)$$

Constants p_i, g_i in formulas (14), (17) and (18) are determined from expressions

$$p_i = \tfrac{1}{2}(2H_{i-1} + h_i)h_i \ ; g_i = \tfrac{1}{3}\left(3H^2_{i-1} + 3H_{i-1}h_i + h_i^2\right) \ h_i \quad (19)$$

In the event that reinforcing of the "j" layer are made with use of geogrid, geonet or geotextiles, it is ne-cessary to make the changes concerning constants $N_{st}, b_{kl}, (s = x, y, xy; k = 1,2,3)$ to formulas (14) and (18):

$$N_{sT} = \sum_{i=1}^{j-1} A^i_{kT} t_{i0} h_i + A^j_{kT} t_{j0} h_j + \sum_{i=j+1}^{m} A^i_{kT} t_{i0} h_i + \sum_{i=1}^{j-1} A^i_{kT} t_{i1} p_i + \sum_{i=j+1}^{m} A^i_{kT} t_{i1} p_i;$$
$$b_{kl} = \sum_{i=1}^{j-1} A^i_{kl} h_i + A^j_{kl} h_j + \sum_{i=j+1}^{m} A^i_{kl} h_i, \quad (20)$$

Substituting Q_x and Q_y from expressions (10) in the third equation (6), we shall receive differential equation of bending of a multilayered plate

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + q = 0. \quad (21)$$

Here

$$q = q_1 + q_2; \ q_1 = q_z^- + Z + \tfrac{\partial}{\partial x} R_x + \tfrac{\partial}{\partial y} R_y;$$
$$q_2 = C_z w - H_m C_z \left(f_x \frac{\partial w}{\partial x} + f_y \frac{\partial w}{\partial y}\right) \quad (22)$$

Parameter q_1 includes the information about external loading, and q_2 – the same about reaction of the elastic basis.

3 LONGITUDINAL - CROSS BENDING OF THE POLYREINFORCED PLATE

3.1 *A conclusion of the differential equation of the bending*

To show the mechanism of reception of the decision of a problem of a longitudinal - cross bending multi-layered constructive - anisotropic plate on the elastic basis we shall consider the elementary case, when in regular intervals distributed loading q_z directed down-wards is applied on all surface of a plate. The plate is in a stationary temperature field, constant in plane XY, and varied piecewise linear in the direction of axis Z according to dependence (5), linear within the limits of each separately taken layer of a plate. As longitudinal we shall choose a directi-on X, and as cross - Y. We shall assume, that the longitudinal edges of a plate parallel to an axis X are free, and cross, parallel axes Y, are rigidly jammed.

Therefore for it the next condition are satisfied

$$\sigma_y = \tau_{xy} = 0. \quad (23)$$

Substituting (23) in (7), we shall receive

$$N_y = N_{xy} = 0. \quad (24)$$

Let's assume, that superficial and volumetric loa-dings X, Y, Z on a plate are absent. We shall reduce a problem of a longitudinal - cross bending of the polyreinforced plate to a problem of a cross bending. For this purpose first of all we shall exclude the tensile and shear strains in expressions (13) and (16). Expressions (13) in view of equality (24) will become

$$
\begin{aligned}
& b_{11}\varepsilon_{ox} + b_{12}\varepsilon_{oy} + b_{13}\gamma_{oxy} + c_{11}\chi_x + c_{12}\chi_y + \\
& + c_{13}\chi_{xy} - N_{xT} = N_x; \\
& b_{21}\varepsilon_{ox} + b_{22}\varepsilon_{oy} + b_{23}\gamma_{oxy} + c_{21}\chi_x + c_{22}\chi_y + \\
& + c_{23}\chi_{xy} - N_{yT} = 0; \\
& b_{31}\varepsilon_{ox} + b_{32}\varepsilon_{oy} + b_{33}\gamma_{oxy} + c_{31}\chi_x + c_{32}\chi_y + \\
& + c_{33}\chi_{xy} - N_{xyT} = 0.
\end{aligned} \tag{25}
$$

In each of the equations (25) we shall unit compo-sed not containing stretching and shear strains

ε_{ox}, ε_{oy}, γ_{oxy}, having designated them according-ly:

N_1, N_2, N_3 :

$$
\begin{aligned}
& N_1 = c_{11}\chi_x + c_{12}\chi_y + c_{13}\chi_{xy} - N_{xT} - N_x; \\
& N_2 = c_{21}\chi_x + c_{22}\chi_y + c_{23}\chi_{xy} - N_{yT}; \\
& N_3 = c_{31}\chi_x + c_{32}\chi_y + c_{33}\chi_{xy} - N_{xyT}.
\end{aligned} \tag{26}
$$

Then expressions (25) will become

$$
\begin{aligned}
& b_{11}\varepsilon_{ox} + b_{12}\varepsilon_{oy} + b_{13}\gamma_{oxy} + N_1 = 0; \\
& b_{21}\varepsilon_{ox} + b_{22}\varepsilon_{oy} + b_{23}\gamma_{oxy} + N_2 = 0; \\
& b_{31}\varepsilon_{ox} + b_{32}\varepsilon_{oy} + b_{33}\gamma_{oxy} + N_3 = 0.
\end{aligned} \tag{27}
$$

System (27) we shall solve concerning deformations.

ε_{ox}, ε_{oy}, γ_{oxy} . Using Kramer's rule, we shall write down;

$$
\varepsilon_{ox} = -\frac{D_1}{D}; \quad \varepsilon_{oy} = -\frac{D_2}{D}; \quad \gamma_{oxy} = -\frac{D_3}{D} \quad ;(28)
$$

Here D, D_1, D_2, D_3 - determinants of matrixes accor-dingly

$$
\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}; \quad
\begin{bmatrix} N_1 & b_{12} & b_{13} \\ N_2 & b_{22} & b_{23} \\ N_3 & b_{32} & b_{33} \end{bmatrix}; \quad
\begin{bmatrix} b_{11} & N_1 & b_{13} \\ b_{21} & N_2 & b_{23} \\ b_{31} & N_3 & b_{33} \end{bmatrix};
$$

$$
\begin{bmatrix} b_{11} & b_{12} & N_1 \\ b_{21} & b_{22} & N_2 \\ b_{31} & b_{32} & N_3 \end{bmatrix}: \tag{29}
$$

From (28) with the account (26) we shall receive

$$
\begin{aligned}
& \varepsilon_{ox} = c^*_{11}\chi_x + c^*_{12}\chi_y + c^*_{13}\chi_{xy} + N^*_1; \\
& \varepsilon_{oy} = c^*_{21}\chi_x + c^*_{22}\chi_y + c^*_{23}\chi_{xy} + N^*_2; \\
& \gamma_{oxy} = c^*_{31}\chi_x + c^*_{32}\chi_y + c^*_{33}\chi_{xy} + N^*_3.
\end{aligned} \tag{30}
$$

Here

$$
\begin{aligned}
& c^*_{11} = c_{11}e_{11} + c_{21}e_{12} + c_{31}e_{13}; \\
& c^*_{12} = c_{12}e_{11} + c_{22}e_{12} + c_{32}e_{13}; \\
& c^*_{13} = c_{13}e_{11} + c_{23}e_{12} + c_{33}e_{13}; \\
& c^*_{22} = c_{12}e_{21} + c_{22}e_{22} + c_{32}e_{23}; \\
& c^*_{23} = c_{13}e_{21} + c_{23}e_{22} + c_{33}e_{23}; \\
& c^*_{33} = c_{13}e_{31} + c_{23}e_{32} + c_{33}e_{33};
\end{aligned} \tag{31}
$$

$$
\begin{aligned}
& N^*_1 = N_x e_{11} + N_{xT} e_{11} + N_{yT} e_{12} + N_{xyT} e_{13}; \\
& N^*_2 = N_x e_{21} + N_{xT} e_{21} + N_{yT} e_{22} + N_{xyT} e_{23}; \\
& N^*_3 = N_x e_{31} + N_{xT} e_{31} + N_{yT} e_{32} + N_{xyT} e_{33}.
\end{aligned}
$$

Here

$$
\begin{aligned}
& e_{11} = \frac{1}{D}(b_{23}b_{32} - b_{22}b_{33}); \quad e_{12} = \frac{1}{D}(b_{12}b_{33} - b_{13}b_{32}); \\
& e_{13} = \frac{1}{D}(b_{22}b_{13} - b_{12}b_{23}); \quad e_{22} = \frac{1}{D}(b_{31}b_{13} - b_{11}b_{33}); \\
& e_{23} = \frac{1}{D}(b_{11}b_{23} - b_{13}b_{21}); \quad e_{33} = \frac{1}{D}(b_{21}b_{12} - b_{11}b_{22}); \\
& e_{21} = e_{12}; \quad e_{31} = e_{13}; \quad e_{32} = e_{23}.
\end{aligned} \tag{32}
$$

Substituting (30) in (16), we shall receive

$$
\begin{aligned}
& M_x = d^*_{11}\chi_x + d^*_{12}\chi_y + d^*_{13}\chi_{xy} + M^*_1; \\
& M_y = d^*_{21}\chi_x + d^*_{22}\chi_y + d^*_{23}\chi_{xy} + M^*_2; \\
& M_{xy} = d^*_{31}\chi_x + d^*_{32}\chi_y + d^*_{33}\chi_{xy} + M^*_3.
\end{aligned} \tag{33}
$$

Here

$$
\begin{aligned}
& d^*_{11} = d_{11} + c_{11}c^*_{11} + c_{12}c^*_{21} + c_{13}c^*_{31}; \\
& d^*_{12} = d_{12} + c_{11}c^*_{12} + c_{12}c^*_{22} + c_{13}c^*_{32}; \\
& d^*_{13} = d_{13} + c_{11}c^*_{13} + c_{12}c^*_{23} + c_{13}c^*_{33}; \\
& d^*_{22} = d_{22} + c_{21}c^*_{12} + c_{22}c^*_{22} + c_{23}c^*_{32}; \\
& d^*_{23} = d_{23} + c_{21}c^*_{13} + c_{22}c^*_{23} + c_{23}c^*_{33}; \\
& d^*_{33} = d_{33} + c_{31}c^*_{13} + c_{32}c^*_{23} + c_{33}c^*_{33}.
\end{aligned} \tag{34}
$$

$$
\begin{aligned}
& M^*_1 = c_{11}N^*_1 + c_{12}N^*_2 + c_{13}N^*_3 - M_{xT}; \\
& M^*_2 = c_{21}N^*_1 + c_{22}N^*_2 + c_{23}N^*_3 - M_{yT}; \\
& M^*_3 = c_{31}N^*_1 + c_{32}N^*_2 + c_{33}N^*_3 - M_{xyT}.
\end{aligned}
$$

Thus, instead of two groups of the equations (12) and (15) we come to one group (33). It allows to reduce a problem of a longitudinal - cross bending of a polireinforsed plate to a problem of a cross bending. Substituting equations (33) in the differential equation of a bend of a plate (21), we shall receive

$$d_{11}^{*}\frac{\partial^4 w}{\partial x^4}+\lambda_1\frac{\partial^4 w}{\partial x^3\partial y}+\lambda_2\frac{\partial^4 w}{\partial x^2\partial y^2}+$$
$$+\lambda_3\frac{\partial^4 w}{\partial x\partial y^3}+d_{22}^{*}\frac{\partial^4 w}{\partial y^4}+\lambda_4\frac{\partial w}{\partial x}- \qquad (35)$$
$$-\lambda_5\frac{\partial w}{\partial y}+C_z w+q_z^{-}=0.$$

Here

$$\lambda_1=d_{13}^{*}+2d_{31}^{*};\ \ \lambda_2=d_{12}^{*}+2d_{33}^{*}+d_{21}^{*};$$
$$\lambda_3=d_{23}^{*}+2d_{32}^{*};\ \ \lambda_4=f_x C_z\left(c_{11}^{*}-H_m\right); \qquad (36)$$
$$\lambda_5=f_y C_z H_m$$

As we deal with a cylindrical bending of a plate in the differential equation (35) all derivatives on y we believe equal to zero. Besides for simplification of the decision as a first approximation we believe equal to zero factor of friction $f_x=0$. Having divi-ded all staying composed on d_{11}^{*}, we shall receive the following equation:

$$\frac{d^4 w}{dx^4}+k^4 w=q^{*}. \qquad (37)$$

Here

$$k^4=\frac{C_z}{d_{11}^{*}};\ \ q^{*}=-\frac{q_z^{-}}{d_{11}^{*}}. \qquad (38)$$

The equation (37) is the non-linear differential equation of the fourth order.

3.2 *The decision of the differential equation of a bending of the polyreinforced plate*

For the decision of the differential equation we search on A.N.Krylov's named method or initial parameters the method:

$$w(x)=C_1F_1(x)+C_2F_2(x)+C_3F_3(x)+$$
$$+C_4F_4(x)+\int_0^x q^{*}F_4(x-u)du \qquad (39)$$

Here $F_1(x),...,F_4(x)$ - are the .Krylov's functi-ons; $C_1,...,C_4$ - the constants of integration expressed through initial parameters

$$C_1=w(0),\ \ C_2=w'(0),\ \ C_3=-\frac{M_x(0)}{d_{11}^{*}},$$
$$C_4=-\frac{Q_x(0)}{d_{11}^{*}} \qquad (40)$$

Regional conditions of a problem:

$$w(0)=0,\ \ w'(0)=0,\ \ w(L)=0,\ \ w'(L)=0. \qquad (41)$$

Unknown initial parameters

$M_x(0)$, $Q_x(0)$ we find from last two regional conditions (41): The maximal deflection we shall define(determine) from expression

$$M_x(0)=-\frac{q_z^{-}}{F_3^2(L)-F_2(L)F_4(L)}\times$$
$$\times\left\{\frac{F_3(L)}{k^4}\left[1-F_1(L)\right]-F_4^2(L)\right\}$$
$$Q_x(0)=-\frac{q_z^{-}}{F_3^2(L)-F_2(L)F_4(L)}\times$$
$$\times\left\{F_3(L)F_4(L)-\frac{F_2(L)}{k^4}\left[1-F_1(L)\right]\right\}$$

(42)

The maximal deflection we shall determine from expression

$$w_{max}=-M_x(0)F_3(L/2)-Q_x(0)F_4(L/2)-$$
$$-\frac{q_z^{-}}{k^4}\left[1-F_1(L/2)\right] \qquad (43)$$

3.3 *A numerical example*

Let's consider a four-layers plate which each layer is executed from isotropic material with the following parameters:

$$h_1=0{,}06m;\ \ E_1=2400MPa;\ \ \nu_1=0{,}25;$$

Layer 1: $A^{(1)}=\begin{bmatrix}2560 & 640 & 0\\ 640 & 2560 & 0\\ 0 & 0 & 960\end{bmatrix}$;

$$h_2=0{,}08m;\ \ E_2=1400MPa;\ \ \nu_2=0{,}25;$$

Layer 2: $A^{(2)}=\begin{bmatrix}1493{,}3 & 373{,}3 & 0\\ 373{,}3 & 1493{,}3 & 0\\ 0 & 0 & 933{,}3\end{bmatrix}$

$$h_3=0{,}3m;\ \ E_3=350MPa;\ \ \nu_3=0{,}25;$$

Layer 3: $A^{(3)}=\begin{bmatrix}373{,}3 & 93{,}3 & 0\\ 93{,}3 & 373{,}3 & 0\\ 0 & 0 & 233{,}3\end{bmatrix}$

$$h_4=0{,}15m;\ \ E_4=40MPa;\ \ \nu_4=0{,}35;$$

Layer 4: $A^{(4)}=\begin{bmatrix}45{,}58 & 15{,}95 & 0\\ 15{,}95 & 45{,}58 & 0\\ 0 & 0 & 14{,}8\end{bmatrix}$

Components of matrixes $A^{(i)}$ for everyone of "i" layer are calculated under formulas of the flat intense condition:

$$A_{11}^{(i)}=A_{22}^{(i)}=\frac{E_i}{1-\nu_i^2};$$
$$A_{12}^{(i)}=A_{21}^{(i)}=\nu_i A_{11}^{(i)};\ \ A_{33}^{(i)}=\frac{E_i}{2(1+\nu_i)}. \qquad (44)$$

Let's set the following numerical values of factor of bed, intensity of the distributed loading and lengths of a plate:

$$C_z = 5\text{MPa}/\text{m};\ \ q_z^- = 60\text{KPa};\ \ L = 4\text{m}.$$

Let's receive

$$\beta = 0{,}5411\ \ \text{m}^{-1};\ \ w_{max} = 23{,}291\times10^{-4}\text{m}.$$

Let's make changes to conditions of a problem. We shall consider the same problem under condition of reinforcing the fourth layer by the cellular confined system having rhombic form cells with diagonals, equal 0,2 m and thickness of walls of 0,001 m.

Characteristics of a ground are

$$E_4 = 40\text{MPa};\ \ \nu_4 = 0{,}35.$$

The module of elasticity of a cellular confined system is $E = 393$МПа.

Matrix $A^{(4)}$ of the fourth reinforced layer it is determined by a technique stated in work of

Nemirovsky (2002):

$$A^{(4)} = \begin{bmatrix} 46{,}324 & 17{,}113 & 0 \\ 17{,}113 & 46{,}324 & 0 \\ 0 & 0 & 15{,}98 \end{bmatrix}$$

As a result of calculations we shall receive

$$d_{11}^{*} = 14{,}623\ \ \text{MNm};\ \ \beta = 0{,}5407\ \ \text{m}^{-1};$$

$$w_{max} = 23{,}235\times10^{-4}\ \ \text{m}.$$

Comparing values of the maximal deflections of two designs, we see, that in the second design they are lower, than in the first. In figure 2 the diagram constructed on the basis of the numerical data, received is given as a result of the decision of a problem at different values of factor of bed.

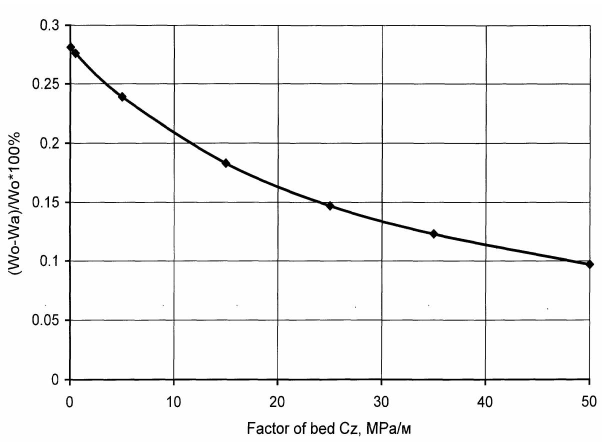

Fig. 2. Dependence reinforcing effect from rigidity of the basis

From the diagram follows, that influence of reinfor-cing of the fourth layer by a cellular confined system on rigidity of all design in the greater degree is shown on the weak bases and tends to decrease at increase of rigidity of the basis.

4 CONCLUSION

The stated theory of calculation of multilayered plates on the elastic basis with the reinforced layers enables the account of various kinds of reinforcing at calculation of designs on the elastic basis. It shows, that the behaviour of a design depends on character reinforcing structures, properties reinforcing materials, rigidity of the basis. Varying these parameters, it is possible to predict behaviour of a design and also to create designs with preset properties.

заранее заданными свойствами.

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