

Permeability of geotextile and granular filters

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ABSTRACT: Relationships derived from the classical Kozeny-Carman's equation show that nonwoven geotextile filters are generally more permeable than granular filters that have the same filtration opening size. The ratio between nonwoven geotextile filter permeability and granular filter permeability is approximately of the order of 10 in many typical cases. Therefore, a nonwoven geotextile filter should be generally preferred to a granular filter in applications where filter permeability is essential. Relationships based on Kozeny-Carman's equation for permeability and Giroud's retention criterion show that nonwoven geotextile filters and granular filters that have the maximum opening size allowed by the retention criterion for a given soil are several orders of magnitude more permeable than that soil. This shows that the permeability criterion for filters is automatically met with a large margin if the filter opening size is close to the maximum value allowed by the retention criterion.

1 INTRODUCTION

By combining the classical Kozeny-Carman's equation, which gives the hydraulic conductivity of a porous medium as a function of its specific surface area, and a relationship developed by the senior author between nonwoven geotextile filtration opening size and fiber diameter (and using the classical relationship between specific surface area and fiber diameter), an original relationship has been established between the hydraulic conductivity and the filtration opening size of a nonwoven geotextile filter. Also, by combining Kozeny-Carman's equation and a relationship developed by the senior author between the specific surface area of a granular medium and the coefficient of uniformity of its particle size distribution curve, an original relationship has been established between the hydraulic conductivity and the coefficient of uniformity of a granular medium. Numerical examples show that there is a good agreement between the hydraulic conductivities predicted using these relationships and the measured hydraulic conductivities of typical nonwoven geotextiles and granular media.

The relationships for hydraulic conductivity mentioned above were combined with the retention criterion for geotextile and granular filters developed by the senior author. The equations thus obtained give the ratio between the hydraulic conductivities of the filter and the retained soil when the filter (geotextile or granular) has the maximum opening size allowed by the retention criterion. Numerical calculations performed with these equations show that: the hydraulic conductivity of a nonwoven geotextile filter that has the maximum opening size allowed by the retention criterion is typically three to four orders of magnitude greater than the hydraulic conductivity of the retained soil; and this ratio is generally smaller for granular filters than for nonwoven geotextile filters, which means that nonwoven geotextile filters are generally more permeable than granular filters. These considerations are useful in applications where the hydraulic conductivity of the filter should be as high as possible to minimize the risk of filter clogging. The paper describes how the proposed methodology has been used to evaluate arbitrary requirements found in the literature, which are intended to maximize the permeability of the selected filter, such as "the hydraulic conductivity ratio between the filter and the soil must be at least 10 or 100" or "the filter opening size must be greater than 3 times the d_{15} of the soil".

2 THEORETICAL EXPRESSION FOR HYDRAULIC CONDUCTIVITY

2.1 Basic equation

The classical Kozeny-Carman's equation for the hydraulic conductivity of porous media is (Carman 1937):

$$k = \left(\frac{\beta \rho g}{\eta} \right) \frac{n^3}{(1-n)^2} \left(\frac{1}{S} \right)^2 \quad (1)$$

where β = dimensionless factor; ρ = liquid density; g = acceleration due to gravity; η = liquid viscosity; n = porosity of the porous medium; and S = specific surface area of the porous medium. The value of 0.1 can be used for β (Giroud 1996). Other numerical values are: $\rho = 1,000 \text{ kg/m}^3$ for water; $g = 9.81 \text{ m/s}^2$; and $\eta = 1 \times 10^{-3} \text{ kg m}^{-1} \text{ s}^{-1}$ for water.

2.2 Granular materials

In this paper, granular materials are assumed to have a particle size distribution curve that is linear in the classical p -log d axes. Hereinafter, this particle size distribution curve will be referred to as "linear particle size distribution curve". The equation of a linear particle size distribution curve is:

$$p = \frac{0.5}{\ln(C'_u)} \ln \left(\frac{d}{d_0} \right) \quad (2)$$

where p = fraction by mass of particles smaller than d ; d = particle size (equivalent diameter); d_0 = smallest particle size in the considered linear particle size distribution curve; and C'_u = linear coefficient of uniformity defined as follows (Giroud 1982):

$$C'_u = \frac{d_{50}}{d_0} = \frac{d_{60}}{d_{10}} = \frac{d_{70}}{d_{20}} = \frac{d_{80}}{d_{30}} = \frac{d_{90}}{d_{40}} = \frac{d_{100}}{d_{50}} = \sqrt{\frac{d_{100}}{d_0}} \quad (3)$$

Useful relationships based on Equation 2 are:

$$d_{10} = (C'_u)^{0.2} d_0 \quad (4)$$

$$d_{15} = (C'_u)^{0.3} d_0 \quad (5)$$

$$d_{85} = (C'_u)^{1.4} d_{15} \quad (6)$$

The specific surface area of a particle is equal to the surface area of the particle divided by the volume of the particle. The specific surface area of a set of particles is equal to the total surface area of all particles of the set divided by the total volume of all particles of the set. These definitions lead logically to two lemmas. First lemma: the specific surface area of a set of equal particles is equal to the specific surface area of any particle of the set. Second lemma: the specific surface area of a set of N subsets of equal particles is the weighted average of the specific areas of the subsets, weighted with the total volume of particles in each subset. If all particles have the same density, the average can be weighted using the mass of each subset. In the case of a linear particle size distribution curve, the mass fraction of particles having a diameter between d and $d+dd$ is dp , which is the derivative of p , obtained from Equation 2:

$$dp = \frac{0.5}{\ln(C'_u)} \frac{dd}{d} \quad (7)$$

The specific surface area of a spherical particle with a diameter d is:

$$S = \frac{6}{d} \quad (8)$$

Based on the first lemma, the subset of particles having a diameter between d and $d+dd$ has the specific surface area expressed by Equation 8. Based on the second lemma, the specific surface area of the entire set of particles is obtained from the specific surface area expressed by Equation 8 weighted using dp and integrating, hence:

$$S = \int_{d_0}^{d_{100}} \left(\frac{6}{d} \right) \frac{0.5}{\ln(C'_u)} \frac{dd}{d} = \frac{3}{\ln(C'_u)} \left(\frac{1}{d_0} - \frac{1}{d_{100}} \right) \quad (9)$$

Combining Equations 3 and 9 gives:

$$S = \frac{3}{d_0 \ln(C'_u)} \left[1 - \frac{1}{(C'_u)^2} \right] \quad (10)$$

A limited series expansion of Equation 10 shows that S is equal to $6/d_0$ for $C'_u = 1$, which is consistent with Equation 8 since $d_0 = d$ for $C'_u = 1$.

Combining Equations 1, 4 and 10 gives the following expression for the hydraulic conductivity of a granular material:

$$k = \left(\frac{1}{9} \right) \left(\frac{\beta \rho g}{\eta} \right) \frac{n^3}{(1-n)^2} \left\{ \frac{\ln(C'_u)}{(C'_u)^{0.2} \left[1 - \frac{1}{(C'_u)^2} \right]} \right\}^2 (d_{10})^2 \quad (11)$$

Combining Equations 1, 5 and 10 gives the following expression for the hydraulic conductivity of a granular material:

$$k = \left(\frac{1}{9} \right) \left(\frac{\beta \rho g}{\eta} \right) \frac{n^3}{(1-n)^2} \left\{ \frac{\ln(C'_u)}{(C'_u)^{0.3} \left[1 - \frac{1}{(C'_u)^2} \right]} \right\}^2 (d_{15})^2 \quad (12)$$

Equations 11 and 12 are equivalent for a granular material having a linear particle size distribution curve. Equation 11 is useful for making comparisons with the following empirical equation proposed by Hazen (1911) for sands with a rather uniform particle size distribution curve (i.e. small value of C'_u):

$$k = C_H (d_{10})^2 \quad (13)$$

where $4000 \text{ m}^{-1} \text{ s}^{-1} \leq C_H \leq 12,000 \text{ m}^{-1} \text{ s}^{-1}$ (based on published values). Calculations performed with Equation 11 using the numerical values given after Equation 1 give values of $k/(d_{10})^2$ ranging between 2000 and 16,000 for $0.25 \leq n \leq 0.35$ and $2 \leq C'_u \leq 5$. This good agreement between Equations 11 and 13 justifies the model used herein. Also, there is a good agreement between values of k obtained using Equation 12 and results of 250 hydraulic conductivity tests reported by Luettich et al. (1992).

2.3 Nonwoven geotextiles

The specific surface area of a filament with a diameter d_f is:

$$S = \frac{4}{d_f} \quad (14)$$

Combining Equations 1 and 14 gives the following expression for the hydraulic conductivity of a nonwoven geotextile:

$$k_{GT} = \left(\frac{1}{16} \right) \left(\frac{\beta \rho g}{\eta} \right) \frac{n_{GT}^3}{(1-n_{GT})^2} (d_f)^2 \quad (15)$$

where n_{GT} = porosity of nonwoven geotextile.

Numerical calculations performed with Equation 15 using the numerical values given after Equation 1 give $1 \times 10^{-3} \leq k_{GT} \leq 9 \times 10^{-3} \text{ m/s}$ for typical needle-punched nonwoven geotextiles characterized by $0.85 \leq n_{GT} \leq 0.92$ and $25 \mu\text{m} \leq d_f \leq 35 \mu\text{m}$. These hydraulic conductivity values are in good agreement with actual values measured on needle-punched nonwoven geotextiles, which justifies the model used herein.

3 RELATIONSHIP BETWEEN HYDRAULIC CONDUCTIVITY AND FILTRATION OPENING SIZE

3.1 Granular filters

As indicated by Giroud (1996) the following approximate value can be used for the filtration opening size of a granular filter, based on work by Kenney et al. (1985):

$$O_F = \frac{d_{15}}{5} \quad (16)$$

where d_{15} is the d_{15} of the granular filter.

Combining Equations 12 and 16 gives the following expression for the hydraulic conductivity of a granular filter:

$$k_{GRA} = \left(\frac{25}{9} \right) \left(\frac{\beta \rho g}{\eta} \right) \frac{n_{GRA}^3}{(1-n_{GRA})^2} \frac{(O_F)^2}{\left\{ \frac{(C'_{uGRA})^{0.3}}{\ln(C'_{uGRA})} \left[1 - \frac{1}{(C'_{uGRA})^2} \right] \right\}^2} \quad (17)$$

where n_{GRA} = porosity of granular filter; and C'_{GRA} = linear coefficient of uniformity of granular filter.

3.2 Nonwoven geotextile filters

In the case of nonwoven geotextiles, the following relationship exists (Giroud 1996):

$$O_F = d_f \left[\frac{1}{\sqrt{1-n_{GT}}} - 1 + \frac{10 n_{GT}}{(1-n_{GT}) \left(\frac{t}{d_f} \right)} \right] \quad (18)$$

where t = thickness of nonwoven geotextile.

Combining Equations 15 and 18 gives the following expression for the hydraulic conductivity of a nonwoven geotextile filter:

$$k_{GT} = \left(\frac{1}{16} \right) \left(\frac{\beta \rho g}{\eta} \right) \frac{n_{GT}^3}{(1-n_{GT})^2} \frac{(O_F)^2}{\left[\frac{1}{\sqrt{1-n_{GT}}} - 1 + \frac{10n_{GT}}{(1-n_{GT}) \left(\frac{t}{d_f} \right)} \right]^2} \quad (19)$$

3.3 Discussion

Equations 17 and 19 show that the hydraulic conductivity of a filter (granular or geotextile) is reduced by a factor N^2 if its opening size is reduced by a factor N if all other parameters remain the same.

From Equations 17 and 19, the hydraulic conductivity ratio between a nonwoven geotextile filter and a granular filter having the same filtration opening size is:

$$\frac{k_{GT}}{k_{GRA}} = \left(\frac{9}{400} \right) \frac{\frac{n_{GT}^3}{(1-n_{GT})^2} \left\{ \frac{(C'_{uGRA})^{0.3} \left[1 - \frac{1}{(C'_{uGRA})^2} \right]}{\ln(C'_{uGRA})} \right\}}{\frac{n_{GRA}^3}{(1-n_{GRA})^2} \left\{ \frac{1}{\sqrt{1-n_{GT}}} - 1 + \frac{10n_{GT}}{(1-n_{GT}) \left(\frac{t}{d_f} \right)} \right\}} \quad (20)$$

Numerical calculations performed with Equation 20 give a wide range of values for the k_{GT}/k_{GRA} ratio. These calculations show that, generally, a nonwoven geotextile filter is more permeable than a granular filter having the same filtration opening size. The calculations show that the k_{GT}/k_{GRA} ratio depends more on the parameters related to the granular filter (n_{GRA} and C'_{uGRA}) than the parameters related to the nonwoven geotextile filter (n_{GT} and t/d_f). The k_{GT}/k_{GRA} ratio is between 5 and 10 for typical nonwoven geotextile filters ($0.85 \leq n_{GT} \leq 0.92$, and $100 \leq t/d_f \leq 200$) if two typical granular filters are considered (one defined by $n_{GRA} = 0.25$ and $C'_{uGRA} = 5$; the other defined by $n_{GRA} = 0.35$ and $C'_{uGRA} = 1$).

4 RELATIONSHIP BETWEEN FILTER HYDRAULIC CONDUCTIVITY AND SOIL CHARACTERISTICS

4.1 Retention criterion

As indicated by Giroud (2002), a retention criterion applicable to all types of filters is expressed as follows:

$$O_F \leq \omega (C'_{us})^{0.3} d_{85S} \quad \text{for } C'_{us} \leq 3 \quad (21)$$

$$O_F \leq \omega \frac{9}{(C'_{us})^{1.7}} d_{85S} \quad \text{for } C'_{us} \geq 3 \quad (22)$$

where ω = dimensionless factor that depends on soil density ($\omega = 1$ for loose soil, 1.5 for medium dense soil, and 2 for dense soil); d_{85S} = soil particle size such that 85% by mass of the soil particles are smaller than d_{85S} ; and C'_{us} = linear coefficient of uniformity of the soil.

4.2 Granular filters

Combining Equation 6, Equation 12 (with n_S for soil porosity and C'_{us} for the coefficient of uniformity of the soil) and Equa-

tions 6, 17 and 21 gives the following expression for the ratio between the hydraulic conductivity of a granular filter that has the maximum opening size allowed by the retention criterion (i.e. Equation 21 with the = sign) and the hydraulic conductivity of the retained soil for $C'_{us} \leq 3$:

$$\frac{k_{GRA}}{k_S} = (25\omega^2) \frac{\frac{n_{GRA}^3}{(1-n_{GRA})^2} \left\{ \frac{\ln(C'_{uGRA})}{(C'_{uGRA})^{0.3} \left[1 - \frac{1}{(C'_{uGRA})^2} \right]} \right\}}{\frac{n_S^3}{(1-n_S)^2} \left\{ \frac{\ln(C'_{us})}{(C'_{us})^2 - 1} \right\}} \quad (23)$$

Combining Equation 6, 12, 17 and 22 gives the following expression for k_{GRA}/k_S in the case where $C'_{us} \geq 3$:

$$\frac{k_{GRA}}{k_S} = (2025\omega^2) \frac{\frac{n_{GRA}^3}{(1-n_{GRA})^2} \left\{ \frac{\ln(C'_{uGRA})}{(C'_{uGRA})^{0.3} \left[1 - \frac{1}{(C'_{uGRA})^2} \right]} \right\}}{\frac{n_S^3}{(1-n_S)^2} \left\{ \frac{\ln(C'_{us})}{1 - \frac{1}{(C'_{us})^2}} \right\}} \quad (24)$$

Numerical calculations performed with Equations 23 and 24 for $\omega = 2$ (the most typical case) show that granular filters that have the maximum opening size allowed by the retention criterion are typically between two and three orders of magnitude more permeable than the retained soil.

4.3 Nonwoven geotextile filters

Combining Equation 12 (with n_S for soil porosity and C'_{us} for the coefficient of uniformity of the soil), Equation 19, and Equation 21 gives the following expression for the ratio between the hydraulic conductivity of a nonwoven geotextile filter that has the maximum opening size allowed by the retention criterion (i.e. Equation 21 with the = sign) and the hydraulic conductivity of the retained soil for $C'_{us} \leq 3$:

$$\frac{k_{GT}}{k_S} = \left(\frac{9}{16} \right) \frac{\frac{n_{GT}^3}{(1-n_{GT})^2} \left\{ \frac{\omega}{\sqrt{1-n_{GT}}} - 1 + \frac{10n_{GT}}{(1-n_{GT}) \left(\frac{t}{d_f} \right)} \right\}}{\frac{n_S^3}{(1-n_S)^2} \left\{ \frac{\ln(C'_{us})}{(C'_{us})^2 - 1} \right\}} \quad (25)$$

Combining Equation 12, 19 and 22 gives the following expression for k_{GT}/k_S in the case where $C'_{us} \geq 3$:

$$\frac{k_{GT}}{k_S} = \left(\frac{729}{16} \right) \frac{\frac{n_{GT}^3}{(1-n_{GT})^2} \left\{ \frac{\omega}{\sqrt{1-n_{GT}}} - 1 + \frac{10n_{GT}}{(1-n_{GT}) \left(\frac{t}{d_f} \right)} \right\}}{\frac{n_S^3}{(1-n_S)^2} \left\{ \frac{\ln(C'_{us})}{1 - \frac{1}{(C'_{us})^2}} \right\}} \quad (26)$$

Numerical calculations performed with Equations 25 and 26 for $\omega = 2$ (the most typical case) show that nonwoven geotextile

filters that have the maximum opening size allowed by the retention criterion are typically between three and four orders of magnitude more permeable than the retained soil.

4.4 Discussion of permeability criterion

The permeability criterion consists of specifying a minimum required value for the ratio between the hydraulic conductivities of the filter and the retained soil. Based on the results of numerical calculations mentioned in Sections 4.2 and 4.3, it appears that the specified ratio should not be greater than 100 for granular filters and 1000 for geotextile filters. Beyond these values, the filter could be unable to meet the retention criterion.

5 RELATIONSHIP BETWEEN FILTRATION OPENING SIZE AND SOIL PARTICLE SIZE

The following requirement, intended to minimize the risk of filter clogging, is sometimes mentioned for the selection of geotextile filters (Holtz et al. 1997):

$$O_F \geq 3 d_{15S} \quad (27)$$

Combining Equations 6 and 21 or 22 gives, respectively:

$$O_F \leq \omega (C'_{uS})^{1.7} d_{15S} \quad \text{for } C'_{uS} \leq 3 \quad (28)$$

$$O_F \leq \omega \frac{9}{(C'_{uS})^{0.3}} d_{15S} \quad \text{for } C'_{uS} \geq 3 \quad (29)$$

Numerical calculations performed with Equations 28 and 29 for $\omega = 2$ (the most typical case) show that the condition expressed by Equation 27 is met for soils having $1.3 < C'_{uS} < 300$, i.e. virtually all soils, if the = sign is used in Equations 28 and 29. Therefore, if a filter has the maximum opening size allowed by the retention criterion, its opening size is generally more than three times greater than the d_{15} of the retained soil. The maximum value of the O_F/d_{15S} ratio is obtained for $C'_{uS} = 3$. This maximum value is 12.9 for $\omega = 2$, 9.7 for $\omega = 1.5$, and 6.5 for $\omega = 1$. The difference between these values and 3 is not large. Therefore, the requirement expressed by Equation 27 is equivalent to saying that the filter opening size must be close to the maximum value allowed by the retention criterion. In conclusion, the requirement expressed by Equation 27 is appropriate, but it is redundant with the requirement that the filter opening size be as large as allowed by the retention criterion, or the requirement that the hydraulic conductivity ratio between the filter and the soil be high. In fact, the hydraulic conductivity ratio between the filter and the soil may be required to be as high as 100 in the case of granular filters and 1000 in the case of geotextile filters, as shown in Section 4.4.

6 CONCLUSIONS

6.1 Summary of equations

The following equations were presented: (1) Equations 11 and 12 giving the hydraulic conductivity of granular materials; (2) Equation 15 giving the hydraulic conductivity of nonwoven geotextiles; (3) Equation 17 giving a relationship between hydraulic conductivity of granular filters and their opening size; (4) Equation 19 giving a relationship between hydraulic conductivity of nonwoven geotextile filters and their opening size; (5) Equation 20 giving the hydraulic conductivity ratio between nonwoven geotextile and granular filters having the same opening size; (6) Equations 23 and 24 giving the hydraulic conductivity ratio between a granular filter and the soil when the granular filter has the maximum opening size allowed by the retention criterion for this soil; and (7) Equations 25 and 26 giving the hydraulic con-

ductivity ratio between a nonwoven geotextile filter and the soil when the nonwoven geotextile filter has the maximum opening size allowed by the retention criterion for this soil.

6.2 Summary of results

Based on the equations summarized above, the following results were established: (1) typical nonwoven geotextile filters are approximately one order of magnitude more permeable than granular filters that have the same opening size; (2) granular filters that have the maximum opening size allowed by the retention criterion are typically between two and three orders of magnitude more permeable than the retained soil; (3) nonwoven geotextile filters that have the maximum opening size allowed by the retention criterion are typically between three and four orders of magnitude more permeable than the retained soil; and (4) the requirement expressed by Equation 27 is approximately equivalent to requiring that the filter opening size be as large as allowed by the retention criterion.

6.3 Discussion

Filters that have the maximum opening size allowed by the retention criterion are several orders of magnitude more permeable than the soil (two to three orders of magnitude in the case of granular filters and three to four orders of magnitude in the case of nonwoven geotextile filters). Therefore requirements often found in the geosynthetic literature that the ratio between filter hydraulic conductivity and soil hydraulic conductivity be greater than 10 or 100 can be met if the filter openings are equal to or close to the maximum opening size allowed by the retention criterion. However, if filter openings are smaller than the maximum opening size allowed by the retention criterion, then the ratio between filter hydraulic conductivity and soil hydraulic conductivity can be significantly reduced, as shown in Section 3.3. Thus, if a filter that has the maximum allowed opening size is 100 times more permeable than the soil, a filter with opening size equal to one third the maximum allowed value will be approximately 10 times more permeable than the soil instead of 100. Therefore, if a high hydraulic conductivity ratio is required between the filter and the soil (for example to minimize the risk of filter clogging) it is important to select a filter with openings as close as possible to the maximum value allowed by the retention criterion. From this viewpoint, a nonwoven geotextile filter is safer than a granular filter because it is typically one order of magnitude more permeable. Clearly, a nonwoven geotextile filter should be generally preferred to a granular filter having the same filtration opening size in applications where filter permeability is essential.

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