Prestigious Lecture

Development of criteria for geotextile and granular filters

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ABSTRACT: A rational development of criteria for geotextile and granular filters is presented. It is shown that, whereas two criteria are sufficient for granular filters, a permeability criterion and a retention criterion, four criteria are required for geotextile filters. The four criteria are: permeability criterion, retention criterion, porosity criterion and thickness criterion. The analysis shows that the permeability criterion includes two requirements, a pore water pressure requirement and a flow rate requirement. It is shown that, in the case of granular filters, the two requirements generally reduce themselves to the classical Terzaghi's permeability criterion, whereas, in the case of geotextile filters, the hydraulic gradient in the soil next to the filter determines which of the two requirements is the most stringent. Regarding the retention criterion, the analysis shows that, for both geotextile and granular filters, a complete retention criterion should take into account the density of the soil and the coefficient of uniformity of its particle size distribution curve. This analysis explains the limitations of the classical Terzaghi's retention criterion. The retention criterion proposed herein provides a means to overcome these limitations. Then, the need for two additional criteria for geotextile filters is pointed out. These two criteria are a porosity criterion expressed as a minimum porosity of the filter and a thickness criterion expressed as a minimum thickness of the filter. It is shown that these two criteria are always met by granular filters and, therefore, are needed only for geotextile filters.

1 INTRODUCTION

1.1 *The Terzaghi Lecture*

This paper is closely based upon the "Terzaghi Lecture" titled "Criteria for geotextile and granular filters" delivered by the author of this paper for the American Society of Civil Engineers (ASCE). This lecture was first presented at the annual meeting of the ASCE Geo-Institute in 2008 in New Orleans. Since then, it has been presented at various locations and it is scheduled to be presented as a "Prestigious Lecture" at the 9th International Conference on Geosynthetics in May 2010 in Guarujá, Brazil.

The lecture and this paper summarize research on filter criteria conducted periodically over thirty years by the author of this paper. However, this paper does not include the calculations supporting the demonstrations presented. Also, no references are provided at the end of this paper. The calculations and references will be presented in a detailed paper to be published in the Journal of Geotechnical and Geoenvironmental Engineering of the ASCE. Essentially, this paper is intended to be read easily, just as one listens to a lecture.

1.2 *Scope of this paper*

While it is recognized that work on filter criteria has been published by many authors, the scope of this paper is limited to a presentation of the work done on this subject by the author of this paper. This work is essentially theoretical. This does not mean that the author of this paper ignored experimental work. In fact, experimental data were used in the development of the work presented herein. However, by just doing rational analyses, it has been possible for the author of this paper to build a coherent system of filter criteria. The presentation of the development of this coherent system provides an opportunity to better understand the mechanisms of filtration. Furthermore, the coherent system of filter criteria presented herein has many practical applications, and it is beneficial to geotechnical engineers to be aware of these applications.

This work was undertaken to improve criteria for geotextile filters, but it is applicable to granular filters as well. In fact, the first steps of the development of criteria for geotextile filters were inspired by granular filter criteria. As a result, this paper often refers to Terzaghi's criteria for granular filters.

1.3 *Terzaghi's filter criteria*

The classical Terzaghi's criteria for granular filters are expressed by the following two equations:

$$d_{15F} \geq 4 \text{ or } 5\ d_{15S} \quad (1)$$

$$d_{15F} \leq 4 \text{ or } 5\ d_{85S} \quad (2)$$

where: d_{15F} = d_{15} of the filter; d_{15S} = d_{15} of the soil; and d_{85S} = d_{85} of the soil. (d_x is defined as the particle size such that the soil contains x% by mass of particles smaller than d_x.)

Equation 1 means that the d_{15} of the filter must not be too small. This is the permeability criterion. Equation 2 means that the d_{15} of the filter must not be too large. This is the retention criterion. The difference between the factors 4 and 5 used in the equations is not significant. For convenience and consistency, the factor 5 will be used in all the discussions presented in this paper.

It should be noted that both of Terzaghi's criteria are expressed using the d_{15} of the granular filter. This is possible because the d_{15} of a granular material (whether it is a filter or not) is related to the size of its openings. An approximate relationship is:

$$\text{Opening size} \approx d_{15} / 5 \quad (3)$$

Equation 3 is based on theoretical and experimental work on arrangements of particles by Silveira, Wittmann, Witt, Kenney, and others.

The opening size of a filter (any type of filter) is defined as the diameter of the largest sphere that can pass through the filter.

1.4 *Examples of early geotextile filter criteria*

In the 1970s, several different criteria for "filter fabrics" were proposed, some of them inspired by Terzaghi's criteria for granular filters. Here is an example of a set of criteria used in the USA in the early 1970s for filter fabrics:

$$k_F > k_S \quad (4)$$

$$0.150 \text{ mm} < O_F < d_{85S} \quad (5)$$

$$4\% < A_R < 36\% \quad (6)$$

where: k_F = hydraulic conductivity of the filter fabric; k_S = hydraulic conductivity of the soil; O_F = filter fabric opening size; and A_R = relative open area of the filter fabric (area of openings/total area of filter fabric).

Early on, it was explicitly mentioned in some US filter criteria that nonwoven fabrics should not be used as filters. In any case, many nonwoven fabrics were eliminated by the requirement that the opening size should be greater than 0.150 mm (Equation 5). (This requirement was even O_F > 0.250 mm in an early stage of filter fabric criteria development.) Furthermore, all nonwoven fabrics were eliminated by the requirement that the relative open area (interpreted as porosity for nonwoven fabrics) should be less than 36% (Equation 6).

Progressively, the anti-nonwoven clauses disappeared and here is an example of the resulting filter fabric criteria:

$$k_F > k_S \quad (7)$$

$$O_F < d_{85S} \quad (8)$$

$$A_R > 4\% \quad (9)$$

However, these criteria for filter fabrics were still unsatisfactory, as will be discussed herein.

The terminology "geotextile" appeared in 1977, and was universally adopted. It will, therefore, be used in the rest of this paper.

1.5 *Additional criteria*

Terzaghi's criteria for granular filters (Section 1.3) comprise two criteria, the permeability criterion and the retention criterion. But, two additional criteria are needed for geotextile filters.

Indeed, since there are almost unlimited possibilities in geosynthetic manufacturing, it is possible to imagine geosynthetic materials that would be unlikely to perform properly as filters (as shown in Sections 4 and 5), such as: geosynthetics with very few openings, and some extremely thin nonwoven geotextiles. Criteria are needed to prevent the use as filters of such materials.

Therefore, for geotextiles to perform properly as filters, two additional criteria are needed: a criterion to ensure that the number of openings is sufficient; and a criterion to ensure that the filter thickness is sufficient. It will be seen that the criterion that ensures a sufficient number of openings is, in fact, a porosity criterion. Accordingly, the next four sections address the permeability criterion, the retention criterion, the porosity criterion, and the thickness criterion.

1.6 *Applicability and limitations*

Terzaghi's criteria for granular filters are applicable only to cohesionless soils. Similarly, the analyses and discussions presented in this paper for geotextile filters are only applicable to cohesionless soils, as usually defined in geotechnical engineering.

Filter criteria, as presented in this paper, are limited to the filtration function. These criteria are only a part of the design and selection of a filter material. Accordingly, the reader of this paper should not expect to find herein any consideration or recommendation outside the strict domain of filter criteria.

Thus, survivability requirements (including installation damage resistance requirements and strength requirements to ensure that the filter resists various stresses) and durability requirements for filters are not addressed in this paper because, strictly speaking, they are not filter criteria, as discussed in Section 6.1. However, these requirements are important and they must be included in filter selection criteria and filter specifications.

It will also be seen in the following section that there are three types of filter application from the viewpoint of retention, and that this paper is devoted to only one of the three types of filter application.

1.7 *Retention modes*

As suggested by Fry, three types of filter application can be considered from the viewpoint of retention:

- Filters providing *total retention* are used in zoned dams on the downstream side of clay zones. Their function is to prevent loss of clay by stopping particles that may migrate as a result of internal erosion of the clay due to piping, cracking or dispersion of the clay. These filters become progressively clogged by the particles they stop; but, in this case, clogging is not detrimental because these filters are part of a system whose function is to retain water. These filters should have openings sufficiently small to retain all particles likely to migrate. As a result, they have a low permeability and they slow down the flow if the clay zone becomes too permeable.
- Filters providing *optimum retention* are the typical filters used in geotechnical structures, such as drainage trenches and blankets in a variety of applications, and downstream drains in dams. These filters must function for the design life of the structure. Therefore, they must remain permeable. Their goal is to retain the soil as a whole but not necessarily all particles. The design of these filters involves adequate balance between permeability and soil retention. Such design is delicate and this is the subject of this paper.
- Filters providing *partial retention* are used in bank protection systems where they are subjected to turbulent, intermittent, and multidirectional flow. Such filters must remain unclogged to prevent instability in case of rapid drawdown. To that end, they typically allow flushing of some soil particles when they are washed by the flow. Thus, progressive erosion of the soil occurs but is slowed down by well designed filters. Bank protection structures may require periodic maintenance.

This paper is exclusively devoted to filters providing optimum retention. These filters are assumed to be subjected to non-turbulent flow in one direction and they are intended to function permanently. Total and partial retention are not considered herein.

2 PERMEABILITY CRITERION

2.1 *The two requirements*

The presence of a filter disturbs the flow of water in the soil upstream of the filter. The selected filter must be such that the disturbance is small and acceptable. The disturbance can affect the pore pressure and the flow rate. Therefore, the permeability criterion includes two requirements: a pore pressure requirement and a flow rate requirement. These two requirements are discussed below.

2.2 *The pore pressure requirement*

As shown in Figures 1 to 3, the presence of a filter potentially increases the pore water pressure in the soil upstream of the filter. A high pore water pressure can have detrimental effects. The selected filter should be such that the pore pressure increase is as small as possible, ideally zero.

Figure 1 shows the steady-state curve of water pressure as a function of depth for the case of water flowing vertically through soil without a filter.

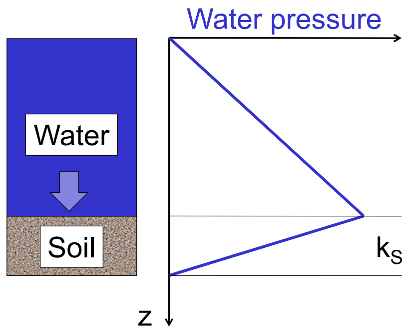


Figure 1. Pore water pressure as a function of depth for the case without a filter.

Figure 2 shows the curve of water pressure as a function of depth for the case where there is a filter. It appears in Figure 2 that there is an excess pore water pressure with respect to the case where there is no filter. This is the general case. However, increasing values of the hydraulic conductivity of the filter, k_F , result in decreasing values of the excess pore water pressure, until an ideal situation is reached where there is no excess pore water pressure, as shown in Figure 3.

An analysis of the pore water pressure curve shown in Figure 3 shows that the presence of the filter causes no pore water pressure increase if the following condition is met:

$$k_F \geq i_S \, k_S \quad (10)$$

where: i_S = hydraulic gradient in the soil next to the filter

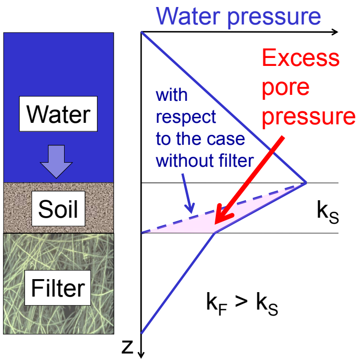


Figure 2. Pore water pressure as a function of depth for the case with a filter. (Note: filter thickness is exaggerated for clarity.)

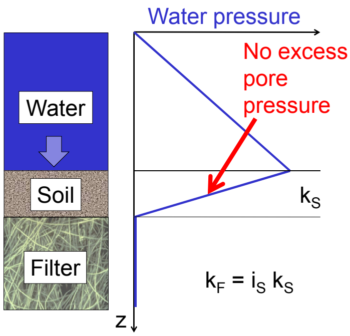


Figure 3. Pore water pressure as a function of depth for the special case with no excess pressure.

Table 1 shows some typical values of the hydraulic gradient in soil next to filters. The values are typically less than 2, except in the case of filters adjacent to clay layers; and they rarely exceed 20. (The high values of the hydraulic gradient in clay do not have an impact on the filter criteria discussed herein, because the applicability of these criteria is limited to cohesionless soils, as indicated in Section 1.6).

Table 1. Typical values of the hydraulic gradient in soil next to filters (values from various sources).

Application	Hydraulic gradient
Dewatering trench	≤ 1.0
Vertical wall drainage	1.5
Road edge drain	≤ 1.0
Inland waterway protection	≤ 1.0
Landfill drainage layer	1.5
Dam toe drain	2.0
Drain behind dam clay core	3 to > 10
Liquid reservoir with clay liner	> 10

2.3 *The flow rate requirement*

The presence of a filter, even one that is very permeable, decreases the liquid flow rate compared with the case without a filter. Calculations done with Darcy's equation show that the reduction in flow rate is less than 10% of the flow rate without a filter if the following conditions are met:

$$k_F \geq k_S \quad \text{for filter thickness 1 to 10 mm} \tag{11}$$

$$k_F \geq 25\, k_S \text{ for filter thickness 250 to 2500 mm} \tag{12}$$

The 1 to 10 mm thickness condition applies to geotextile filters. Therefore, Equation 11 becomes:

$$k_F \geq k_S \quad \text{for geotextile filters} \tag{13}$$

The 250 to 2500 mm thickness condition applies to granular filters. Therefore, Equation 12 becomes:

$$k_F \geq 25\, k_S \text{ for granular filters} \tag{14}$$

2.4 *Comparison of the two requirements*

The two requirements, the pore water pressure requirement and the flow rate requirement, can be grouped as follows for geotextile filters:

$$k_F \geq \max\left(i_S\, k_S\,,\ k_S\right) \tag{15}$$

and for granular filters:

$$k_F \geq \max\left(i_S\, k_S\,,\ 25\, k_S\right) \tag{16}$$

In the case of geotextile filters, the flow rate requirement ($k_F \geq k_S$) may be more stringent or less stringent than the pore pressure requirement ($k_F \geq i_S\, k_S$) depending on the hydraulic gradient, i_S , in the soil next to the filter.

In the case of granular filters, the flow rate requirement ($k_F \geq 25\ k_S$) is generally more stringent than the pore pressure requirement ($k_F \geq i_S\ k_S$) because the hydraulic gradient, i_S , is generally less than 25.

It is important to note that the flow rate requirement for granular filters ($k_F \geq 25\ k_S$) can be expressed in terms of particle size. Indeed, it is well known that the permeability of a granular material is approximately proportional to the square of a small particle size such as d_{10} or d_{15} :

$$k_F \approx \kappa\, d_{15F}^2 \tag{17}$$

$$k_S \approx \kappa\, d_{15S}^2 \tag{18}$$

where κ is a factor of proportionality.

Equations 17 and 18 are adapted from the classical Hazen's equation, which is a special case of the classical Kozeny-Carman equation.

The value of the factor κ can be assumed to be approximately the same for the soil and the granular filter if it is assumed that the soil and the granular filter have similar particle size distribution curves, which is a typical assumption in granular filter design. At least, the soil and the granular filter should have similar particle size distribution curves over the lower part of the curve, e.g. between d_5 and d_{40} (which is the part of the particle size distribution curve that governs permeability).

Eliminating κ between Equations 14, 17 and 18 shows that Equation 14 is equivalent to:

$$d_{15F} \geq 5\, d_{15S} \tag{19}$$

Equation 19 is the classical Terzaghi's permeability criterion for granular filters (see Equation 1).

The above discussion shows that the classical permeability criterion for granular filters is based only upon the flow rate requirement. In other words, it does not include the pore pressure requirement (but it is generally more stringent).

Therefore, there is a difference between geotextile filters and granular filters regarding the permeability criterion: in the case of geotextile filters, either the pore pressure requirement governs or the flow rate requirement governs (depending on the hydraulic gradient); whereas in the case of granular filters, the flow rate requirement (i.e. Terzaghi's permeability criterion) generally governs.

2.5 *Conclusion on permeability criteria*

The foregoing discussions show that, to develop an appropriate permeability criterion for geotextile filters, it would have been incorrect to simply imitate Terzaghi's permeability criterion for granular filters.

It is true that Terzaghi's permeability criterion provided a starting point for the derivation of a permeability criterion for geotextile filters. However, to extend to geotextile filters the work of Terzaghi on granular filters, it was necessary to thoroughly understand the requirements related to water flow through a filter and the adjacent soil.

3 RETENTION CRITERION

3.1 *Introduction to retention and definitions*

The development of a retention criterion is the most complex part of the development of filter criteria. In fact, it can be argued that retention is the most important aspect of the filtration function. Therefore, it is not surprising that filters can be classified based on the mode of retention (see Section 1.7).

Prior to presenting the development of the proposed retention criterion, it is necessary to present some definitions. It will be shown that the proposed retention criterion depends on the soil density and the soil particle size distribution curve coefficient of uniformity.

The soil density will be expressed herein using the density index, I_D , traditionally called "relative density", defined as follows:

$$I_D = \frac{e_{\max} - e}{e_{\max} - e_{\min}} \tag{20}$$

where: e = void ratio of the soil; $e_{\min}$ = minimum void ratio of the soil; and $e_{\max}$ = maximum void ratio of the soil. Both $e_{\min}$ and $e_{\max}$ are measured using a conventional procedure. The minimum void ratio corresponds to the maximum density of the soil (I_D = 1). The maximum void ratio corresponds to the minimum density of the soil (I_D = 0).

The definition of the coefficient of uniformity is well known:

$$C_u = \frac{d_{60}}{d_{10}} \tag{21}$$

In the analyses performed for the development of equations for the proposed retention criterion, a *linear coefficient of uniformity* is used rather than the coefficient of uniformity in order to obtain simple equations. The linear coefficient of uniformity is based on a straight line that closely follows the central part of the actual particle size distribution curve (Figure 4).

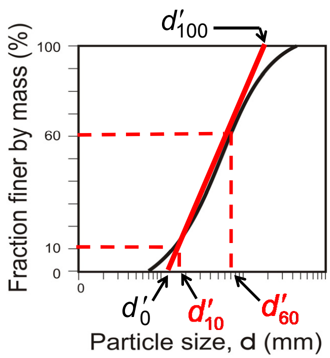


Figure 4. Linear coefficient of uniformity. The straight line follows the central part of the particle size distribution curve.

The linear coefficient of uniformity is defined as:

$$C'_u = \frac{d'_{60}}{d'_{10}} \text{ which is equal to } \sqrt{\frac{d'_{100}}{d'_0}} \tag{22}$$

where d'_x is the "linear particle size" derived from the straight line in Figure 4 (where the subscript x is defined after Equation 2). Obviously, the straight line approximation presented in Figure 4 is possible only if the particle size distribution curve of the soil is continuous, which excludes gap-graded soils.

3.2 *Adaptation of Terzaghi's retention criterion*

How should we select the maximum allowable opening size of a geotextile filter to retain a soil? A simple answer consists in adapting to geotextiles Terzaghi's retention criterion for granular filters (i.e. Equation 2 written with the factor 5):

$$d_{15F} \leq 5\ d_{85S} \qquad (23)$$

Combining Equations 3 and 22 gives:

$$O_F \leq d_{85S} \qquad (24)$$

where O_F (introduced in Section 1.4 as filter fabric opening size) is more generally defined herein as filter opening size, regardless of the type of filter. It should be noted that various symbols (e.g. O_{95} and *AOS*) are used for the opening size of geotextile filters. Such symbols are often linked to a given method of physical measurement of the geotextile filter opening size. A discussion of methods of opening size measurement is beyond the scope of this paper. It is assumed herein that O_F designates the opening size measured using an appropriate method.

Equation 24 means that a filter should only retain large soil particles. Retaining only large soil particles works if the large particles retain smaller particles: in other words, if the soil is internally stable. Therefore, an ideal retention criterion should take into account not only the opening size of the filter, but also the internal stability of the soil.

Thus, Terzaghi's retention criterion is incomplete because it does not take into account the internal stability of the soil retained by the filter. However, Terzaghi's retention criterion has been successfully used for decades to design granular filters.

There are two reasons for the success of Terzaghi's retention criterion in granular filter design.

The first reason is that, to a certain degree, granular filters work, even if the soil lacks internal stability, because they are thick. The mechanism is the following: particles that are not retained may accumulate in the filter, thereby decreasing the filter opening size, until the filter functions. In other words, a granular filter may adapt itself to the soil (to a certain degree).

However, filter thickness alone does not explain the adequacy of Terzaghi's retention criterion for granular filters.

The second, and most important, reason for the success of Terzaghi's retention criterion for granular filters is that its use has been limited to the most stable soils, i.e. soils with a maximum particle size less than a certain value, typically 4.75 mm (i.e. the opening size of the US sieve No. 4); hence, the practice of truncation of the particle size distribution curve, which will be discussed later (see Section 3.8 and the Appendix at the end of this paper).

3.3 *Development of a criterion for geotextile filters*

While granular filters benefit (*to a certain degree*) from their thickness, geotextile filters are thin, which has created an incentive for developing a more accurate retention criterion, a criterion that takes into account the internal stability of the soil.

In internally stable soils, there are particles of a certain size that form a continuous skeleton. This continuous skeleton entraps particles that are a little smaller than the skeleton particles. In turn, these particles entrap particles that are a little smaller, and so on. Therefore, if a filter has openings such that the soil skeleton is retained, then all particles smaller than the skeleton particles are retained (with the exception of a few small particles located between the skeleton and the filter; this is why there are some fine particles in suspension in the water during the first phase of functioning of a filter).

Also, particles larger than the skeleton particles will be retained because, if a filter retains certain particles, it will retain coarser particles. But, if a filter has openings too large to retain the skeleton, the soil is not retained because the particles retained in this case do not form a continuous skeleton.

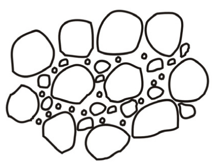
In conclusion, a filter must have openings such that the skeleton is retained. This leads to two questions: What is the size of the skeleton particles? And what is the maximum opening size for a filter to retain a skeleton composed of particles of a given size? (In other words, what is the maximum allowable filter opening size?) These two questions are addressed in the next two sections.

3.4 *Size of the skeleton particles*

As mentioned in Section 3.3, retention of the skeleton is essential for the retention of an internally stable soil. Internal stability depends on the particle size distribution of the soil, which is characterized by the coefficient of uniformity.

Geometric considerations show that, in a soil with a coefficient of uniformity of approximately 3, particles are tightly interlocked. In other words, such a soil has maximum internal stability. This is illustrated in Figures 5, 6 and 7.

As illustrated in Figure 5, when a soil has a coefficient of uniformity *equal to or less than 3*, the coarsest particles form a continuous skeleton that entraps all other particles.



$C_u \lesssim 3$

Figure 5. Schematic representation of a soil with a coefficient of uniformity equal to or less than 3.

As illustrated in Figure 6, when a soil has a coefficient of uniformity *greater than 3*, the coarsest particles are generally not in contact with each other. As a result, they do not form a continuous skeleton that entraps other particles. In other words, when a soil has a coefficient of uniformity *greater than 3*, the coarsest particles "float" in the matrix formed by the other particles. In this case, the coarsest particles do not contribute to internal stability.

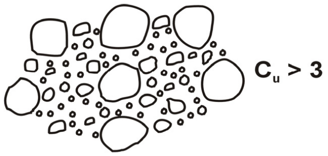


Figure 6. Schematic representation of a soil with a coefficient of uniformity greater than 3.

In the case of a soil having a coefficient of uniformity greater than 3, only the fine fraction of this soil that has a coefficient of uniformity equal to 3 will be taken into account in the development of the retention criterion. In other words, the coarse fraction will be ignored, as illustrated in Figure 7.

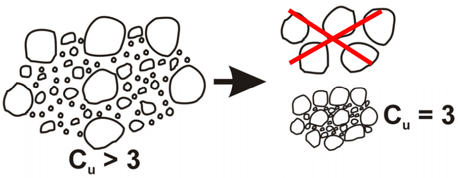


Figure 7. Elimination of the coarsest particles of a soil with a coefficient of uniformity greater than 3, in order to use, in the retention criterion, only the fine fraction with a coefficient of uniformity of 3.

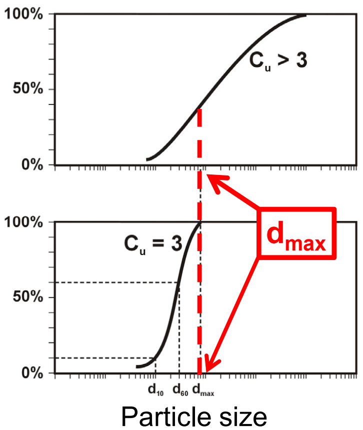


Figure 8. Automatic truncation of the particle size distribution curve of a soil with a coefficient of uniformity greater than 3.

The approach presented in Figure 7 means that, in the development of the proposed retention criterion, the particle size distribution curve is truncated and only the particles smaller than d_{max} are considered (Figure 8). However, as explained below, the truncation is only virtual.

The user of the proposed retention criterion does not have to do the truncation. The truncation is done automatically (i.e. it is included in the equations, given in Section 3.6, that express the retention criterion). As shown in Figure 8, only the fraction of the curve with a coefficient of uniformity of 3 is used in developing the criterion.

3.5 *Maximum allowable filter opening size*

As indicated at the end of Section 3.3, the following question needs to be addressed in developing the retention criterion: What is the maximum opening size for a filter to retain a skeleton composed of particles of a given size? (In other words, what is the maximum allowable filter opening size?) It will be shown that the answer to this question depends on the soil density.

If the soil is in a loose state (represented by a cubic arrangement), all particles pass through the filter if the opening size is larger than the particle size (Figure 9).

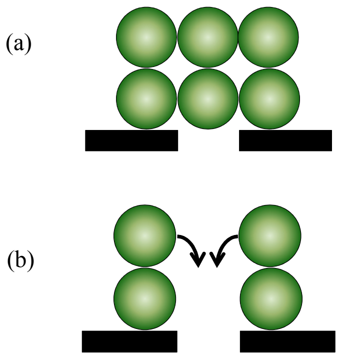


Figure 9. Schematic representation of a loose soil by a cubic arrangement: (a) cubic arrangement on a filter with openings as large as particles; (b) most or all particles pass through the filter.

If the soil is in a dense state (represented by a hexagonal arrangement), particles do not pass through the filter if the opening size is as large as the particle size (Figure 10) because a stable bridge forms as soon as the first particle passes. If the soil is in a dense state, particles pass if the opening size is twice as large as the particle size (Figure 11).

Based on this demonstration, the internal stability of a soil depends not only on its coefficient of uniformity, but also on its density.

(a)

(b)

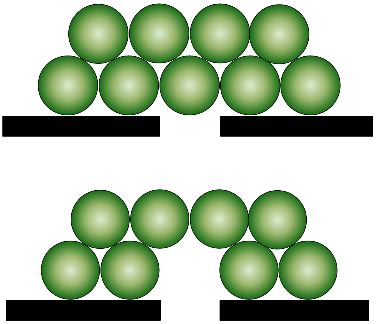


Figure 10. Schematic representation of a dense soil by a hexagonal arrangement: (a) hexagonal arrangement on a filter with openings as large as particles; (b) formation of a stable bridge after one particle has passed through the filter.

(a)

(b)

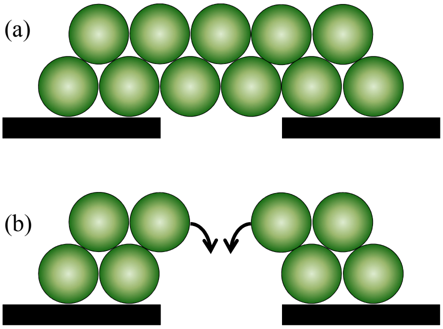


Figure 11. Dense soil represented by a hexagonal arrangement in contact with a filter having openings twice as large as the particles: most or all particles pass.

3.6 *Equations for the proposed retention criterion*

In the preceding sections, the discussion of the retention criterion has been mostly qualitative, but a quantitative analysis of the role of the coefficient of uniformity and the density of soil has been done. A mathematical analysis, not presented herein, leads to the following equations for $C'_u \le 3$:

$$O_F \le (C'_u)^{0.3} \, d'_{85S} \quad \text{for a loose soil} \qquad (25)$$

$$O_F \le 1.5 (C'_u)^{0.3} d'_{85S} \quad \text{for a medium dense soil} \qquad (26)$$

$$O_F \le 2 (C'_u)^{0.3} \, d'_{85S} \quad \text{for a dense soil} \qquad (27)$$

and the following equations for $C'_u \ge 3$:

$$O_F \le \frac{9 \, d'_{85S}}{(C'_u)^{1.7}} \quad \text{for a loose soil} \qquad (28)$$

$$O_F \le \frac{13.5 \, d'_{85S}}{(C'_u)^{1.7}} \quad \text{for a medium dense soil} \qquad (29)$$

$$O_F \le \frac{18 \, d'_{85S}}{(C'_u)^{1.7}} \quad \text{for a dense soil} \qquad (30)$$

where C'_u is the linear coefficient of uniformity defined by Equation 22 and Figure 4; and d'_{85S} is the "linear particle size" of the soil that corresponds to 85% by mass, defined in Section 3.1 after Equation 22. The soil density is evaluated using the density index defined by Equation 20; and the three density categories are as follows: $I_D \ge 65\%$ (dense), $35\% < I_D < 65\%$ (medium dense), and $I_D \le 35\%$ (loose).

It should be noted that Equations 25 to 30 depend on the soil density and the soil linear coefficient of uniformity, which is consistent with the discussions presented in Sections 3.3 to 3.5. However, in the discussions presented in Section 3.3 to 3.5, the coefficient of uniformity is used whereas, in the above equations, the *linear* coefficient of uniformity is used. This is because the mathematical analysis is simpler and leads to simpler equations when performed with the *linear* coefficient of uniformity than the coefficient of uniformity.

3.7 *Graphical representation of the proposed retention criterion*

The equations for the proposed retention criterion are represented graphically in Figure 12 along with Terzaghi's retention criterion, expressed in terms of opening size (Equation 24). All curves have a peak for the most stable soil, i.e. for a soil having a linear coefficient of uniformity of 3.

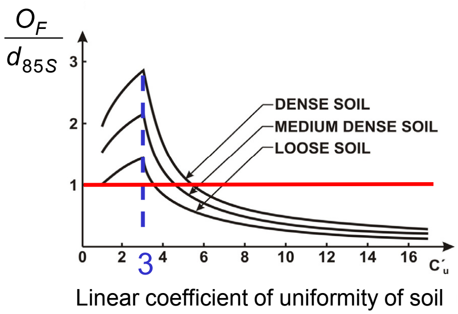


Figure 12. Graphical representation of retention criteria for geotextile filters. The three curves represent the proposed retention criterion (Equations 25 to 30). The horizontal line represents Terzaghi's retention criterion adapted to geotextile filters (Equation 24).

There is a significant difference between the two retention criteria, Terzaghi's criterion and the proposed criterion. In the case of small coefficients of uniformity, the proposed retention criterion allows opening sizes larger than Terzaghi's retention criterion. This is in agreement with experimental data, as mentioned in Section 3.8: therefore, the difference between the proposed criterion and Terzaghi's criterion for small coefficients of uniformity does not mean that the proposed retention criterion is unconservative, and it may mean that there is a risk of clogging if Terzaghi's retention criterion is used for small coefficients of uniformity. For soils having a high coefficient of uniformity, Figure 12 shows that using $O_{F\max} = d_{85S}$ (the horizontal line, i.e. Terzaghi's criterion) allows filter openings that are too large, hence a risk of piping.

Therefore, a geotextile filter is safer if it is designed with a retention criterion that takes into account the internal stability of the soil.

3.8 *Retention criterion for granular filters*

The approach used for geotextile filters in the preceding sections can be used for granular filters. This approach leads to the graph shown in Figure 13 for the proposed retention criterion for granular filters in the case of a dense soil.

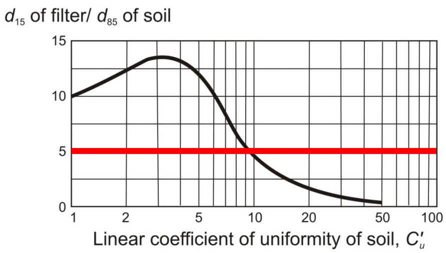


Figure 13. Retention criterion for granular filters in the case of a dense soil. The horizontal line represents Terzaghi's retention criterion for granular filters.

In Figure 13, the vertical axis is d_{15F} / d_{85S} to be consistent with the design practice for granular filters. As a result, Terzaghi's retention criterion is represented by the horizontal line d_{15F} / d_{85S} = 5 (Equation 2).

In Figure 13, the values of d_{15F} / d_{85S} greater than 5 for uniform soils are consistent with experimental data presented by Bertram, Sherard and others. Figure 13 also shows that, for large coefficients of uniformity, Terzaghi's retention criterion is unconservative. It is for this reason that truncation of the particle size distribution curve is traditionally employed in the design of granular filters. Truncation artificially decreases the coefficient of uniformity of the soil to compensate for the fact that Terzaghi's retention criterion is unconservative in the case of high coefficients of uniformity.

The retention criterion for granular filters shown in Figure 13 is applicable regardless of the maximum particle size. In other words, it is not limited to particles smaller than 4.75 mm. Therefore, the need for the potentially inaccurate operation of truncating the particle size distribution curve of the soil at 4.75 mm is eliminated.

In fact, with the proposed retention criterion, truncation is done automatically (i.e. it is included in the equations). More importantly, the automatic truncation takes place at the appropriate location, which is generally *not* 4.75 mm.

Therefore, by extending to granular filters the retention criterion developed for geotextile filters, a new criterion for designing granular filters has been obtained. This criterion is simpler and safer than the traditional criterion in the case of soils having a large coefficient of uniformity.

3.9 *Limitation related to particle size distribution*

The analyses presented in Section 3 and the proposed retention criterion are only applicable to the case where the particle size distribution curve is continuous (as pointed out at the end of Section 3.1). The case of discontinuous (e.g. gap-graded) particle size distribution curves is beyond the scope of this paper. In fact, the same limitation applies to Terzaghi's retention criterion.

4 POROSITY CRITERION

4.1 *The need for an additional criterion*

As indicated in Section 1.5, it is necessary in the case of geotextile filters to have a criterion that ensures that the number of filter openings is sufficient. It will be seen in this section that the criterion that ensures that "*the number of filter openings is sufficient*" results in a porosity criterion.

It should be noted that many geotextiles are so permeable that, even if a geotextile has a small number of openings per unit area, it may still meet the permeability criterion. Clearly, the permeability criterion is not sufficient to eliminate geotextile filters that do not have enough openings. Therefore, a criterion specific to the number of openings per unit area is needed.

4.2 *Disturbance of flow*

Flow of liquids through porous media, such as soils and granular or fibrous filters, takes place in tortuous channels. These channels are called flow channels.

It is possible to demonstrate that the number of flow channels per unit area is greater in the soil than in a filter *that meets the retention criterion for that soil*. As a result, the flow of liquid is disturbed at the soil-filter interface due to flow concentration into a smaller number of flow channels.

Disturbance of the flow at the soil-filter interface could cause displacement of fine soil particles in the vicinity of the filter, which could result in accumulation of fine soil particles at the surface of the filter or inside the filter. This accumulation of fine particles could cause clogging of the soil-filter system. Therefore, the number of flow channels in the filter per unit area should be as large as possible in order to minimize disturbance of the flow of liquid from the soil (retained by the filter) to the filter.

There is an additional reason for the number of flow channels in the filter per unit area to be as large as possible. In spite of the presence of a properly designed filter, there is always the possibility that some fine soil particles will move, in particular during the first phase of functioning of a filter. If such fine particles clog some flow channels in the filter, their impact on flow rate will be reduced if the number of flow channels in the filter per unit area is larger.

4.3 *Determination of the number of openings*

Depending on the filter structure, the number of flow channels is either equal or proportional to the number of filter openings. Therefore, a filter should have as many openings as possible per unit area.

To the best knowledge of the author of this paper, no rational analysis of the required number of filter openings is available and none was developed by the author. The only relevant guidance is that it is known that granular filters work.

Thus, it may be assumed that, if the number of filter openings in a geotextile filter is equal to, or greater than, the number of filter openings in a typical granular filter, then the geotextile filter should work, i.e. should have a sufficient number of openings.

Therefore, the first step of the development of a porosity criterion consists in determining the number of openings in a granular filter. The number of openings depends on the geometric characteristics of the granular filter.

Herein, the number of openings will be expressed as a function of one of the geometric characteristics of the granular filter: the filter opening size. This will make it possible to objectively compare geotextile filters and granular filters having the same opening size.

Calculations (not presented herein) show that the number of openings per unit area in a typical granular filter is approximately given by the following equation:

$$N_o \approx \frac{0.1}{O_F^2} \tag{31}$$

This number of openings per unit area will be used as a minimum value for the number of openings per unit area for geotextile filters.

Calculations (not presented herein) show that the number of openings per unit area for woven geotextiles is given by the following equation:

$$N_o = \frac{A_R}{O_F^2} \tag{32}$$

where: A_R = relative open area (open area/total area) of a woven geotextile (as defined after Equation 6).

Comparing Equations 31 and 32 gives the following required value for the relative open area of a woven geotextile filter:

$$A_R \geq 0.1 \tag{33}$$

Equation 33 means that the relative open area of a woven geotextile filter should be equal to or greater than 0.1 to ensure that the number of openings in the woven geotextile filter is at least equal to the number of openings in a granular filter with the same opening size.

In the case of nonwoven geotextile filters, the determination of the number of openings per unit area is difficult. Only an approximate calculation has been done, and only lower and upper boundaries were obtained. Here are the lower and upper boundaries for the number of openings per unit area in nonwoven geotextile filters, N_o:

$$\frac{\left(1-\sqrt{1-n}\right)^2}{O_F^2} \leq N_o \leq \frac{4\left(1+0.4n-\sqrt{1-n}\right)^2}{\sqrt{3}\ O_F^2} \tag{34}$$

where: n = porosity of the nonwoven geotextile.

Comparing Equations 31 and 34 and performing calculations for a wide range of values of the porosity, n, give a conservative criterion, which ensures that the number of openings in the nonwoven geotextile is at least equal to the number of openings in a granular filter having the same opening size. The conservative criterion is that the porosity of a nonwoven geotextile filter should be equal to or greater than 0.55.

In conclusion, the two criteria are:

for woven geotextiles $A_R \geq 0.1$ (i.e. 10%)

and

for nonwoven geotextiles $n \geq 0.55$ (i.e. 55%)

4.4 *Discussion*

The relative open area of a woven geotextile (A_R) is the two-dimensional equivalent of porosity (n). Therefore, the two criteria can be grouped under the terminology "porosity criteria". Since the criteria for the two types of geotextile filters are of the same nature, one may wonder why the numerical values are so different: 0.10 for woven geotextile filters and 0.55 for nonwoven geotextile filters.

The reason is that, in woven geotextile filters, most of the pore space is used for flow, whereas, in nonwoven geotextile filters, a significant fraction of the pore space is not used for flow. The case of granular filters is intermediate, because particles occupy a larger fraction of the pore space than fibers of a nonwoven geotextile. As a result, there is less space that is *not* used for flow in a granular filter than in a nonwoven geotextile filter.

In summary, among filters having the same opening size, the same number of openings per unit area is achieved by:

- woven geotextile filters with a relative open area (i.e. *two-dimensional porosity*) of 10%,
- granular filters with a porosity of 20-30%, and
- nonwoven geotextile filters with a porosity of 55%.

The porosity criterion has never been formulated for granular filters because all granular filters have approximately the same porosity (20-30%). In contrast, the porosity criterion is important for geotextile filters, because there is a wide range of porosities, from 0.01 for some woven geotextiles to 0.92 for some nonwoven geotextiles.

Woven geotextiles with a relative open area less than 0.1 should not be used as filters, as shown above. However, many of the available woven geotextiles have a relative open area less than 0.1 and some of them are used as filters. These filters have a high risk of clogging, as discussed in Section 4.2.

All nonwoven geotextiles (even when subjected to compressive stresses, which decrease their porosity) meet the porosity criterion ($n \geq 0.55$), because their porosity is typically 0.7-0.9 (uncompressed) and 0.6-0.8 (compressed). In fact, in the case of nonwoven geotextile filters, the porosity criterion should be applied to uncompressed geotextiles. This is because a compressive stress normal to the plane of a nonwoven geotextile does not change the number of its openings, and the calculations mentioned in Section 4.3 were performed for a quasi-isotropic geotextile, which is representative of the uncompressed state of a nonwoven geotextile. However, one may consider it more conservative to use the porosity criterion with the porosity of the geotextile filter in a compressed state in the field.

5 THICKNESS CRITERION

5.1 *The need for an additional criterion*

As indicated in Section 1.5, it is necessary, in the case of nonwoven geotextile filters, to have a criterion that ensures that the thickness of the filter is sufficient. The discussion of filter thickness will address both nonwoven geotextile filters and granular filters. Woven geotextile filters will not be considered because thickness is not relevant in their case (as explained at the end of Section 5.2).

5.2 *Filtration paths and constrictions*

To understand the role of filter thickness, it is necessary to understand how a soil particle travels through a filter. A soil particle that travels through a filter (any filter, granular or geotextile) must go through passages called constrictions (a terminology proposed by Kenney.)

In the case of a geotextile filter, a constriction is a passage between fibers. The size of a constriction can be defined as the diameter of the largest sphere that passes through the constriction (Figure 14).

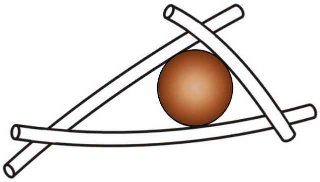


Figure 14. Constriction (i.e. passage between fibers) in a geotextile filter and spherical particle just passing through the constriction, which defines the constriction size.

A soil particle that travels through a filter moves from one constriction to another, thereby following a filtration path, which is identical to a flow channel (see Section 4.2). The particle will be stopped or will pass depending on the size of constrictions along the filtration path. As shown in Figure 15, a particle can be stopped on top of the filter or inside the filter, or can pass through the filter.

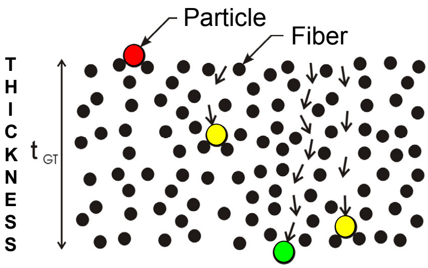


Figure 15 Schematic cross section of a nonwoven geotextile filter showing one particle stopped on top of the geotextile, two particles stopped inside the geotextile, and one particle passing through the geotextile filter.

It will be shown in Section 5.5 that the reliability of a filter is a function of the number of constrictions in any given filtration path. In the case of woven geotextile filters, there is only one constriction in each filtration path. Therefore, changing the thickness of a woven geotextile filter does not affect its reliability. This is why it was stated in Section 5.1 that thickness is not relevant in the case of woven geotextile filters,

5.3 *Definitions of two curves*

Consider the *nonwoven material* used to make a nonwoven geotextile. This material is characterized by a constriction size distribution curve (Figure 16).

An important point on the curve is the smallest constriction size.

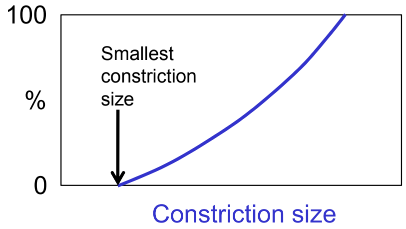


Figure 16. Constriction size distribution curve.

The constriction size distribution curve is a characteristic of the *material* from which the geotextile is made. The constriction size distribution curve is *not* a characteristic of the geotextile. A nonwoven geotextile, i.e. a nonwoven material with a given thickness, is characterized by another curve: the opening size distribution curve (Figure 17).

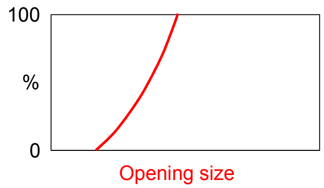


Figure 17. Opening size distribution curve.

One may ask the following question: Why is there a curve and not a single value of the opening size (as for granular filters)? This is a legitimate question because, so far in this paper, each geotextile filter has been characterized by a unique opening size. To answer this question, a demonstration is needed.

Each filtration path contains a number of constrictions. A soil particle can go through the filter if it is smaller than the smallest constriction in a path. Therefore, each filtration path is characterized by the smallest constriction that exists in that path. This is why the lowest point of the curve of Figure 16 is an important point.

Thus, the smallest constriction in a given filtration path is the opening size of that filtration path. In general, the smallest constriction is different in each filtration path. Therefore, each filtration path has a different opening size. As a result, a filter is characterized by an opening size distribution curve.

Knowing that the opening size of a filter is defined as the diameter of the largest sphere that can pass through the filter, it is possible to visualize the opening size of the filter on the opening size distribution curve. The opening size of the filter is naturally the largest opening size of the filtration paths. In other words, the opening size of a filter is the top of the opening size distribution curve (Figure 18). Thus, a filter is characterized by an opening size distribution curve of which the top is the filter opening size.

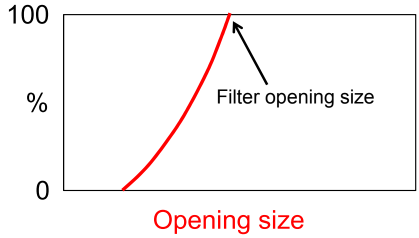


Figure 18. Opening size distribution curve and filter opening size.

5.4 *Relationship between the two curves*

The relationship between the opening size distribution curve (which characterizes a filter with a given thickness) and the constriction size distribution curve (which characterizes the filter material) will be established in this section. To establish this relationship, geometric considerations must be made:

- In a filter (granular or geotextile), there is a very large number of filtration paths (millions per m^2).
- In a nonwoven geotextile filter, due to the limited thickness, there are not many constrictions in each filtration path (10 to 100).

Statistically, the conclusion of the two preceding geometric considerations is that:

- The smallest filtration path opening size is the smallest constriction size.
- The largest filtration path opening size is *not* the largest constriction size.

Accordingly, the two curves have the same origin, but not the same top (Figure 19).

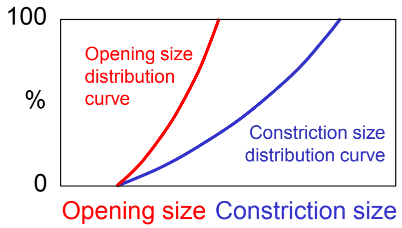


Figure 19. Relationship between constriction size distribution curve and opening size distribution curve.

The analysis that led to Figure 19 may be difficult to understand. It is easier to understand the relationship between the two curves by using limit cases.

The first limit case is the following. If a nonwoven geotextile filter had a quasi-zero thickness, there would be only one constriction per filtration path. Therefore, in this case, the opening size distribution curve would be the same as the constriction size distribution curve (Figure 20).

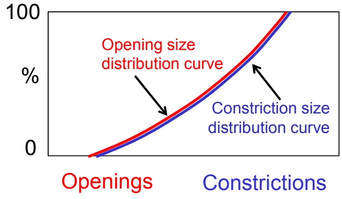


Figure 20. Opening size distribution curve for an infinitely thin geotextile: it is identical to the constriction size distribution curve.

The second limit case is the following. If a filter has an infinite thickness, the probability for having the smallest constriction in every filtration path is 100%. As a result, in that case, all the filtration paths have the same opening size, which is the smallest constriction size. Therefore, in this case, the opening size distribution curve is a vertical line (Figure 21). This vertical line is at the location of the smallest constriction size shown in Figure 16.

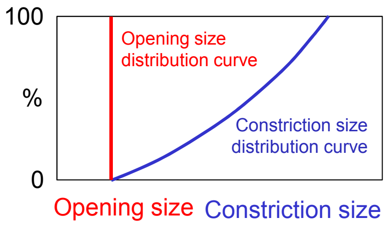


Figure 21. Opening size distribution curve for an infinitely thick geotextile: it is a vertical line passing through the location of the smallest constriction size (see Figure 16).

Typical nonwoven geotextile filters made of the same nonwoven material (i.e. nonwoven geotextiles that differ only by thickness) are between the two limit cases (Figure 22). In addition, since the opening of a filter is the top of the opening size distribution curve, it appears on Figure 22 that, for filters made of the same material, the filter opening size increases with decreasing thickness.

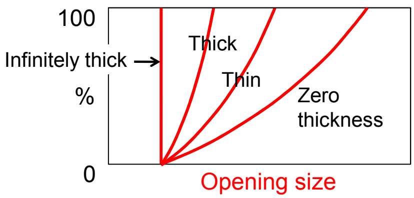


Figure 22. Opening size distribution curve for typical nonwoven geotextiles made from the same nonwoven material characterized by a constriction size distribution curve that is identical to the opening size distribution curve for a nonwoven geotextile with zero thickness.

An important conclusion can be drawn from Figure 22: For a given filter material (characterized by a unique constriction size distribution curve), the opening size of a filter depends on its thickness. In fact, the opening size decreases for increasing values of the thickness of the filter. This is true for all types of filters. However, from a practical standpoint, there is a major difference between nonwoven geotextile filters and granular filters: due to construction reasons, granular filters have a thickness that can be considered quasi-infinite compared to inter-constriction distance. In other words, contrary to geotextile filters, granular filters have a very large number of constrictions in their filtration paths. Therefore granular filters made of the same granular material have an opening size that does not depend on thickness (whereas nonwoven geotextile filters have an opening size that depends on thickness).

5.5 *Quantitative analysis*

In the preceding section, the impact of geotextile thickness on filter opening size has been discussed qualitatively. However, this impact can be evaluated quantitatively. A mathematical analysis has been made, and a relationship has been developed between nonwoven geotextile opening size and nonwoven geotextile thickness:

$$\frac{O_F}{d_f} \approx \frac{1}{\sqrt{1-n}} - 1 + \frac{10\,n}{(1-n)\,t_{GT}\,/\,d_f} \qquad (35)$$

where: O_F = nonwoven geotextile filter opening size; t_{GT} = nonwoven geotextile thickness; and d_f = fiber diameter.

Equation 35 was developed theoretically with the exception of the factor 10, which was obtained through calibration using a large number of experimental data. Equation 35 is represented by a family of curves (Figure 23) that give the ratio of Opening size/Fiber diameter as a function of the ratio of Geotextile thickness/Fiber diameter. This graph is only for nonwoven geotextiles.

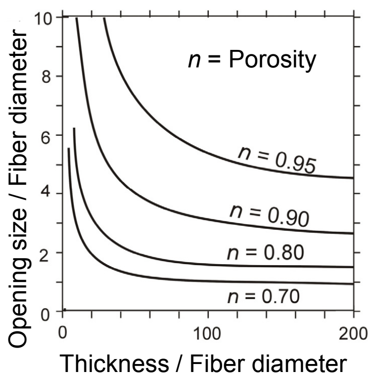


Figure 23. Ratio of Opening size/Fiber diameter as a function of the ratio of Geotextile thickness/Fiber diameter for nonwoven geotextile filters (Equation 35).

It can be seen on Figure 23 that, for a given porosity (i.e. for a given nonwoven material under a given normal stress), the geotextile filter opening size decreases for increasing values of the geotextile thickness. This is consistent with the discussions presented in Section 5.3.

The porosity of a nonwoven geotextile can be calculated using the following equation:

$$n = 1 - \frac{\mu_{GT}}{\rho_f \, t_{GT}} \qquad (36)$$

where: μ_{GT} = mass per unit area of the geotextile; and ρ_f = density of fiber material.

Combining Equations 35 and 36 gives:

$$\frac{O_F}{d_f} \approx \frac{1}{\sqrt{1-n}} - 1 + \frac{10\,n}{\mu_{GT}/\left(\rho_f \, d_f\right)} \qquad (37)$$

Equation 37 can be used rather than Equation 35 when the thickness of the geotextile is not known and the mass per unit area is known.

Based on the same analysis, the number of constrictions through a nonwoven geotextile filter is given by the following approximate equation:

$$N_{constrictions} \approx \frac{\mu_{GT}}{\rho_f \, d_f \sqrt{1-n}} \qquad (38)$$

A parametric study of the curves of Figure 23 (using Equations 35 and 38) has shown that, beyond a geotextile thickness containing approximately 25 constrictions, the opening size is not significantly affected by changes in geotextile thickness. In other words, a geotextile thickness that contains more than 25 constrictions is approximately an infinite thickness from the viewpoint of opening size. The zone of the graph with more than 25 constrictions is limited by a quasi-straight line (Figure 24). In this zone, the curves are relatively flat because they are close to the asymptote, which indicates that variations in geotextile filter thickness do not have a significant impact on filter opening size.

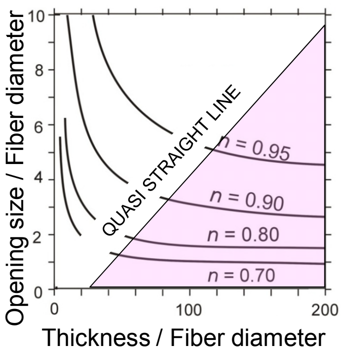


Figure 24. Zone of the graph of Figure 23 with more than 25 constrictions.

Therefore, to be reliable, a nonwoven geotextile filter should have a thickness that corresponds to at least 25 constrictions. This is the thickness criterion. It is convenient to express the thickness criterion in terms of numbers of constrictions, so the criterion is expressed in a way that is independent of the geotextile porosity and the fiber diameter. The ASTM standard D 7178 on filters uses this work.

5.6 *Quantification of the opening size distribution curve*

In conclusion, a nonwoven geotextile filter can be characterized by a unique opening size, the filter opening size. However, only a certain number of filtration paths have this opening size. Other filtration paths have a smaller opening size, as indicated by the opening size distribution curve (Figure 18). Each filtration path in a nonwoven geotextile filter has an opening size that is between the smallest constriction size and the filter opening size, as shown in Figure 25.

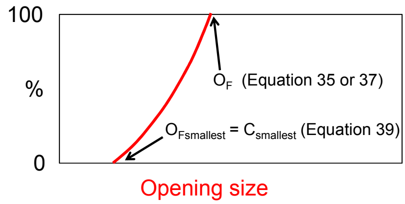


Figure 25. Opening size distribution curve showing the theoretical values of the two extreme points.

Therefore, it is important to quantify the smallest constriction size and the filter opening size. The filter opening size is given by Equation 35 (or 37) and the smallest constriction size, which is also the smallest opening size (according to Figure 19), is given by Equation 35 for infinite thickness (according to Figure 21), hence the following equation derived from Equation 35 with $t_F = \infty$:

$$\frac{C_{smallest}}{d_f} = \frac{O_{Fsmallest}}{d_f} \approx \frac{1}{\sqrt{1-n}} - 1 \qquad (39)$$

where: $C_{smallest}$ = smallest constriction size; and $O_{Fsmallest}$ = smallest opening size.

The quantified opening size distribution curve is represented in Figure 25.

5.7 *Potential limitation of the analysis*

Equation 35 was obtained assuming that a nonwoven geotextile is a slice of homogeneous and isotropic nonwoven material. It may be argued (as pointed out by Palmeira) that a needle-punched nonwoven geotextile may not be considered a homogeneous and isotropic material because of the presence of holes punched by the needles during the manufacturing process. On the other hand, it is possible that, to a certain degree, the factor 10 (which was obtained experimentally, as indicated in Section 5.5) takes into account the presence of the needle holes. Clearly, this is an area where more research work is needed. In the meantime, Equation 35 (and the derived equation, Equation 37) are powerful tools that make it possible to calculate the opening size of a nonwoven geotextile filter and predict changes in opening size caused by changes in porosity, which happens when a geotextile filter is subjected to compressive stresses (see the section titled "*Influence of compressive stress on opening size*" in the Appendix at the end of this paper).

6 CONCLUSION ON CRITERIA

6.1 *Criteria for geotextile filters*

Four filter criteria have been established for geotextile filters: permeability criterion, retention criterion, porosity criterion, and thickness criterion. One may wonder if these four criteria form a complete set of filter criteria.

Filtration is governed by filter openings. The characteristics of filter openings are the size, shape, density (number per unit area) and distribution. The four criteria address three of these four characteristics: the size, density and distribution.

The shape of filter openings is not addressed in the four criteria. The shape of filter openings is likely to be a minor consideration in the case of filters with a quasi-random structure such as granular filters and nonwoven geotextile filters. In contrast, the shape of openings may be a relevant issue in the case of some woven geotextiles and some other types of man-made filters (such as perforated plates).

Clearly, more work is needed on the impact of the shape of filter openings on the performance of filters. In the meantime, the set of four criteria discussed in this paper can be considered complete, at least for granular and nonwoven filters.

Furthermore, the set of four criteria presented in this paper is a coherent set of criteria, because it addresses only parameters that are relevant to the filtration function. Other types of requirements, such as survivability requirements (including installation damage resistance requirements and strength requirements) and durability requirements, which are sometimes presented as part of geotextile filter criteria, are not filter criteria because they are not intrinsic to the filtration function. (In other words, they can be used for geotextiles performing other functions.) It is not because they are not important that these requirements are not considered herein, but because they do not belong to the subject of this paper, the filter criteria.

6.2 *Comparison with criteria for granular filters*

Regarding the permeability criterion, the following have been shown:

- A complete permeability criterion should include two requirements, a pore water pressure requirement and a flow rate requirement.
- The pore water pressure requirement depends on the hydraulic gradient in the soil next to the filter.
- In each specific case, one of these two requirements is more stringent than the other and is, consequently, the governing requirement.
- In the case of geotextile filters, either the pore water pressure requirement governs or the flow rate requirement governs, depending on the hydraulic gradient in the soil next to the filter.
- In the case of granular filters, the flow rate requirement generally governs.
- In the case of granular filters, the flow rate requirement is identical to the classical Terzaghi's permeability criterion.

Regarding the retention criterion, the following have been shown:

- A complete retention criterion should take into account the internal stability of the soil.
- The internal stability of the soil depends on its coefficient of uniformity and its density.
- A retention criterion that depends on the coefficient of uniformity and the density of the soil has been developed.
- This retention criterion developed for geotextile filters has been extended to granular filters.
- In the case of granular filters, this retention criterion is applicable regardless of the coefficient of uniformity of the soil.
- Therefore, unlike the classical Terzaghi's retention criterion (the use of which is limited to soils having a relatively small coefficient of uniformity), the proposed retention criterion can be used regardless of the coefficient of uniformity of the soil.
- As a result, the potentially inaccurate practice of truncating the particle size distribution curve of the soil (which has been used in geotechnical engineering since the 1950s to make it possible to use Terzaghi's retention criterion with soils having a large coefficient of uniformity) is not needed with the proposed retention criterion.

It has also been shown that, in addition to the two classical filter criteria, the permeability criterion and the retention criterion, two other criteria are needed, the porosity criterion and the thickness criterion.

Regarding the porosity criterion, the following have been shown:

- All filters must have a sufficient number of openings per unit area, to minimize flow concentrations at the soil-filter interface, which could cause soil particle displacement and, potentially, clogging of the soil-filter system.

- This requirement can be expressed in the form of a porosity criterion.
- The porosity criterion is expressed as a minimum porosity for nonwoven geotextile filters and a minimum relative open area for woven geotextile filters (the relative open area of a woven geotextile being the two-dimensional equivalent of porosity).
- The porosity criterion is not needed for granular filters because they have sufficient porosity to ensure a sufficient number of openings per unit area to minimize flow concentration.

Regarding the thickness criterion, the following have been shown:

- A soil particle that travels through a filter follows a filtration path and passes through passages called constrictions.
- Two nonwoven geotextile filters made of the same nonwoven material and having different thicknesses have different opening sizes, the greater thicknesses being associated with smaller opening sizes.
- In other words, the opening size of a nonwoven geotextile filter depends on its thickness.
- If a nonwoven geotextile filter is sufficiently thick to contain more than 25 constrictions in its filtration paths, its opening size does not depend significantly on thickness variations. In other words, nonwoven geotextile filters need to be sufficiently thick to be reliable.
- No thickness requirement is needed for granular filters because, due to construction constraints, granular filters are thick and the number of constrictions in a granular filter is very large; therefore, if there was a thickness criterion for granular filters, it would always be met.

7 CASE HISTORY

7.1 Presentation of the case history

The case history discussed in this section has been cited in several publications by various authors. This is the case of the first geotextile filter used in a dam, Valcros Dam, constructed in 1970 in France. A geotextile filter was used under the rip-rap protecting the upstream slope of the dam and, more importantly, a geotextile filter was used in the downstream drain of the dam.

Valcros Dam is a 17 m high homogeneous dam constructed with silty sand including 30% by mass of particles smaller than 0.075 mm. The author of this paper was the design engineer for the dam and he could not get adequate sand for the filter of the downstream drain. Therefore, he elected to use a nonwoven geotextile that had never been used before as a filter. In fact, it is possible that this was the first use of a nonwoven geotextile as a filter in civil engineering.

The construction of the downstream drain of the dam with a geotextile filter is shown in Figure 26. The geotextile used in the downstream drain is a needle-punched nonwoven geotextile, with a porosity of 0.92 and a mass per unit area of 300 g/m^2, made of continuous polyester filaments having a diameter of 27 μm.

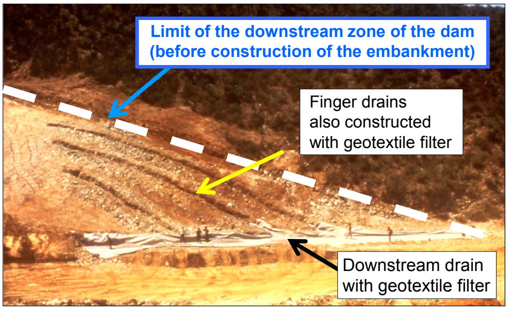


Figure 26. Construction of Valcros Dam downstream drain.

The performance of the drain has been satisfactory since the first filling of the reservoir, which has remained full since then. In particular:

- A constant trickle of clean water has been observed at the drain outlet. There were particles in suspension in the water only for a few days after the filling of the reservoir, which is consistent with the fact that the fine soil particles located between the soil skeleton and the filter must pass through the filter, as pointed out in Section 3.3.
- The flow rate at the drain outlet has been consistent with the hydraulic conductivity of the embankment soil ($k_s \approx 1 \times 10^{-7}$ m/s).
- No seepage of water has ever been observed through the downstream slope.

Tests performed on samples of geotextile removed from the actual filter 6 years and 22 years after construction gave the following results:

- The geotextile filter was in good mechanical condition. The loss in tensile strength was 10 to 20% between year zero and year 6 and was negligible between years 6 and 22. Based on microscopic observation of the fibers, this loss may be attributed to construction damage.
- The hydraulic conductivity of the geotextile was practically unaffected by soils particles entrapped: the measured hydraulic conductivity was 1×10^{-3} m/s for samples taken from the dam and 1.5×10^{-3} m/s for the same samples after careful washing.
- Clogging of the geotextile has been negligible (0.2% of the pore volume of the geotextile), which confirms the hydraulic conductivity measurements mentioned above.

The outstanding performance of the geotextile filter can be explained. First, it should be noted that, fortunately, the Valcros Dam filter was not designed using criteria derived directly from the classical Terzaghi's filter criteria. In reality, the author of this paper did not use filter criteria to select the geotextile filter for Valcros Dam. The geotextile filter was selected on the basis of limited experimental data available at that time (1970) from the use of this geotextile under an experimental embankment constructed on saturated soft soil. Therefore, some degree of luck was involved in the success of the Valcros Dam filter. This encouraged the author of this paper to work on filter criteria. As a result, geotextile filter criteria can now be used for Valcros Dam.

7.2 *Use of filter criteria for Valcros Dam*

The permeability criterion (Equation 15) is met with a very large factor of safety by the geotextile filter used in Valcros Dam because the ratio between the geotextile filter hydraulic conductivity and the embankment soil hydraulic conductivity is 10,000, based on the values given in Section 7.1: $k_s \approx 1 \times 10^{-7}$ m/s and $k_F = 1 \times 10^{-3}$ m/s (for the sample containing soil particles).

The porosity criterion ($n \geq 0.55$, as indicated in Section 4.3) is easily met by the geotextile filter used in Valcros Dam because its porosity is 0.92 as indicated in Section 7.1.

The thickness criterion is expressed in terms of number of constrictions. As indicated in Section 5.4, the number of constrictions should be greater than 25. The number of constrictions in the geotextile filter used in Valcros Dam can be calculated using Equation 38 as follows (knowing that the density of polyester is 1,380 kg/m^3):

$$N_{constrictions} \approx \frac{0.3}{1{,}380 \times 0.000027\sqrt{1-0.92}} = 28$$

This calculated number of constrictions is larger than 25, but barely. While this is acceptable, it would have been better to use a heavier geotextile of the same type. In fact, the same type of geotextile, but with a mass per unit area of 400 g/m^2, was used under rip-rap for the upstream slope protection of Valcros Dam. If that geotextile had been used as a filter for the downstream drain, the number of constrictions would have been 38.

The retention criterion requires a long discussion. It is addressed in the next two sections.

7.3 *Use of retention criteria for Valcros Dam*

The particle size distribution curve of the Valcros Dam embankment material is shown in Figure 27. It can be derived from the figure that the coefficient of uniformity is 90 (as calculated using Equation 21) and the linear coefficient of uniformity is 53 (as calculated using Equation 22) with the values of d'_0 and d'_{100} found in Figure 27: $d'_{0S} = 0.0075$ mm and $d'_{100S} = 21$ mm.

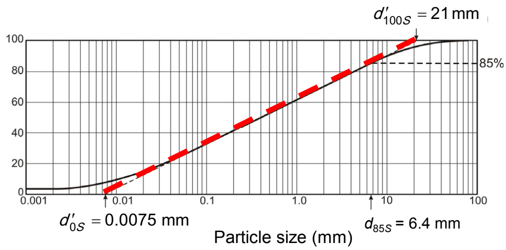


Figure 27. Valcros Dam particle size distribution curve. The dashed straight line that closely follows the central part of the particle size distribution curve is used to determine the linear coefficient of uniformity (see Figure 4 and Equation 22).

First, Terzaghi's retention criterion directly adapted to geotextile filters is used (Equation 24). An allowable opening size of 6.4 mm is obtained (Figure 28). This opening size is very large. Such an opening size does not seem adequate to retain a silty sand. The problem is caused by the large coefficient of uniformity of the silty sand. The problem that results from a large coefficient of uniformity is known by geotechnical engineers. Their traditional solution consists in truncating the particle size distribution curve. The truncated particle size distribution curve is shown in Figure 29.

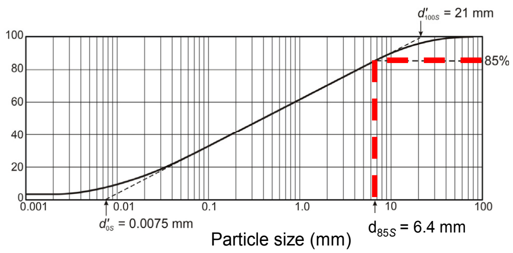


Figure 28. Use of Terzaghi's retention criterion adapted to geotextile filters with Valcros Dam particle size distribution curve.

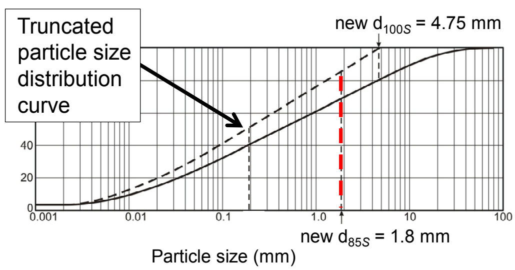


Figure 29. Use of Terzaghi's retention criterion adapted to geotextiles with Valcros Dam truncated particle size distribution curve.

Using Terzaghi's retention criterion adapted to geotextile filters with the truncated particle size distribution curve gives an allowable opening size of 1.8 mm (see the new d_{85S} in Figure 29). This is still very large and does not seem adequate to retain a silty sand.

It should be noted that the retention criterion used above, i.e. Terzaghi's retention criterion, is a criterion for cohesionless soils. This is justified because the soil in Valcros Dam embankment has negligible cohesion. However, since this soil contains 30% particles smaller than 0.075 mm, which may generate some cohesion, it is possible to use a criterion proposed by Sherard. This criterion, intended for cohesive soils, depends on the percentage of particles smaller than 0.075 mm.

Using Sherard's criterion for the soil in Valcros Dam embankment gives an allowable opening size of 2.7 mm. This is very large. Then, Sherard's criterion was used with the truncated particle size distribution curve, and an allowable opening size of 0.8 mm was obtained. This is still very large.

The soil in Valcros Dam embankment seems to defy all retention criteria.

7.4 *Use of the proposed retention criterion*

Finally, the proposed retention criterion is used. From the six equations (Equations 25 to 30) presented in Section 3.6, the equation for dense soil and a coefficient of uniformity greater than 3, i.e. Equation 30, should be used since the soil in the embankment can be assumed to be dense and the linear coefficient of uniformity ($C'_u = 53$) is larger than 3.

With $d'_{85S} = 6.4$ mm and $C'_u = 53$, Equation 30 gives:

$$O_F \le \frac{(18)(6.4)}{(53)^{1.7}} = 0.135 \text{ mm}$$

This value seems reasonable for a silty sand; and it seems more reasonable for a silty sand than the values obtained above: 6.4 mm with Terzaghi's retention criterion; 1.8 mm with Terzaghi's retention criterion applied to the truncated particle size distribution curve; 2.7 mm with Sherard's retention criterion; and 0.8 mm with Sherard's retention criterion applied to the truncated particle size distribution curve.

It is interesting to use the proposed retention criterion with the truncated particle size distribution curve of the soil in the Valcros Dam (even though it can be predicted that this exercise is useless because the truncation is already included in the equation, as indicated in Section 3.4). In this case, the linear coefficient of uniformity is 25 (calculated from Figure 29 using Equation 22) and the d'_{85S} is 1.8 mm (see Figure 29). Equation 30 gives:

$$O_F \le \frac{(18)(1.8)}{(25)^{1.7}} = 0.135 \text{ mm}$$

Comparing the last two calculations, it appears that the same result is obtained with or without truncation of the particle size distribution curve. This remarkable result confirms the validity of the proposed retention criterion.

Clearly, the potentially inaccurate operation of truncation of the particle size distribution curve is not needed with the proposed retention criterion. This has just been shown in the case of a geotextile filter; it could be shown in the case of a granular filter.

In conclusion, the required value of the Valcros Dam geotextile filter opening size is 0.135 mm. This seems more reasonable, for a silty sand, than the values obtained using other criteria: 6.4 mm, 2.7 mm, 1.8 mm and 0.8 mm.

The opening size of the geotextile filter measured on samples taken from the dam is 0.1 mm. The opening size of the Valcros Dam geotextile filter can also be calculated knowing the physical characteristics measured on geotextile samples taken from the dam (fiber diameter = 0.027 mm; and mass per unit area = 0.300 kg/m^2) and knowing the fiber density, 1380 kg/m^3 (density of polyester). Using Equation 37 gives:

$$\frac{O_F}{0.027} \approx \frac{1}{\sqrt{1-0.92}} - 1 + \frac{10 \times 0.92}{0.3 / (1380 \times 0.000027)}$$

hence: $O_F = 0.099$ mm

This is in good agreement with the measured value of 0.1 mm. Therefore, the retention criterion ($O_F \le 0.135$ mm) is met. (If the same type of geotextile with a mass per unit area of 400 g/m^2 had been used, as suggested in Section 7.2, an opening size of 0.092 mm would have been calculated using the above equation.)

7.5 *Conclusion of the case history*

The geotextile filter has performed well since 1970. The four filter criteria are met, which is consistent with the good performance.

8 SUMMARY AND CONCLUSION

8.1 *Summary*

This paper started with a review of the two classical filter criteria, the permeability criterion and the retention criterion, for both granular and geotextile filters. Then, two criteria were added for geotextile filters: the porosity criterion and the thickness criterion.

Original analyses led to analytical expressions and/or graphical representations for the four criteria. These expressions are simple and readily usable by engineers designing filters.

8.2 *Conclusion for filters*

The conclusion for geotextile filters is that the four proposed criteria for geotextile filters form a coherent set of criteria that allow safe design of geotextile filters.

The conclusion for granular filters is that the retention criterion for granular filters can be improved based on developments made for the retention criterion of geotextile filters.

8.3 *General conclusion*

What started as technology transfer from geotechnical engineering to geosynthetics engineering has resulted in technology transfer from geosynthetics engineering to geotechnical engineering.

Terzaghi would have certainly agreed that his famous filter criteria were not frozen forever, but, rather, could lead to new developments. And Terzaghi would have certainly agreed that, with a new filter material, the geotextile, it was not sufficient to simply adapt his criteria, but it was necessary to review the fundamentals of filtration, and develop new criteria.

Just imitating the great masters is not the best approach to solving modern problems. We do not have to do today what Terzaghi would have done 50 years ago. We need to do today what Terzaghi would do today.

9 ACKNOWLEDGEMENTS

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10 NOTATIONS

A_R	=	relative open area (area of openings/total area of filter);
$C_{smallest}$	=	smallest constriction size;
C_u	=	coefficient of uniformity;
C'_u =	=	linear coefficient of uniformity;
d_{15F}	=	d_{15} of the filter;
d_{15S}	=	d_{15} of the soil;
d_{85S}	=	d_{85} of the soil;
d_f	=	fiber diameter;
d_x	=	particle size such that the soil contains x% by mass of particles smaller than d_x ;
d'_x	=	"linear particle size", i.e. d_x measured on the straight line that runs along the central portion of the particle size distribution curve;
I_D	=	density index (also called relative density);
i_S	=	hydraulic gradient in the soil next to the filter;
k_F	=	hydraulic conductivity of the filter;
k_S	=	hydraulic conductivity of the soil;
N_o	=	number of openings per unit area of filter;
$N_{constrictions}$	=	number of constrictions along a filtration path;
n	=	porosity of filter
O_F	=	filter opening size;
$O_{Fsmallest}$	=	smallest opening size;
t_{GT}	=	nonwoven geotextile thickness;
μ_{GT}	=	mass per unit area of the geotextile; and
ρ_f	=	density of fiber material.

Basic SI units are: $C_{smallest}$ (m), d (m), d' (m), k (m s^{-1}), N_o (m^{-2}), O_F (m), t_{GT} (m), μ_{GT} (kg m^{-2}), and ρ_f (kg m^{-3}). Other symbols are dimensionless.

APPENDIX

Discussion of truncation

As indicated in Section 3.8, the practice of truncation of the particle size distribution curve of the soil is intended to make it possible to use Terzaghi's retention criterion (Equation 2) with soils having a large coefficient of uniformity. Traditionally, truncation takes place at 4.75 mm. Thus, geotechnical engineers ignore soil particles larger than 4.75 mm when they design filters using Equation 2.

The value of 4.75 mm is arbitrary: it corresponds to the US Standard Sieve No. 4, which is often used as the upper limit for sand particles. One may wonder how such an arbitrary practice has been successful. As explained below, the truncation at 4.75 mm may work when the soil to be retained does not contain a large amount of fines (i.e. particles smaller than 0.075 mm).

As shown in Figure 30, a particle size distribution curve with a maximum particle size of 4.75 mm has a coefficient of uniformity of approximately 5 if the percentage of fines is zero (according to Equation 21 with $d_{60S} \approx 0.7$ mm and $d_{10S} \approx 0.14$ mm) and has a coefficient of uniformity of approximately 10 if the soil contains 10% fines after truncation (according to Equation 21 with $d_{60S} \approx 0.7$ mm and $d_{10S} = 0.075$ mm). As seen in Figure 13, in the 5-10 range for the linear coefficient of uniformity (assumed to be close to the coefficient of uniformity), Terzaghi's retention criterion is below the proposed retention criterion. Therefore, in this range, Terzaghi's retention criterion can be used with no risk of piping. In other words, for a soil containing less than 10% fines after the particle size distribution curve has been truncated at 4.75 mm, it is approximately safe to use Equation 2 (i.e. Terzaghi's retention criterion).

The explanation presented above should not be construed as a general justification of the truncation method. In the case of soils containing more than 10% fines, the truncation method leads to excessively large values of the filter opening size. This was illustrated in Section 7.3 for the case of Valcros Dam, where the soil contained 29% fines before truncation and 36% after truncation (see Figure 29): Equation 24 (which is derived from Equation 2) gave a filter opening size of 1.8 mm after truncation, whereas the correct filter opening size was 0.135 mm.

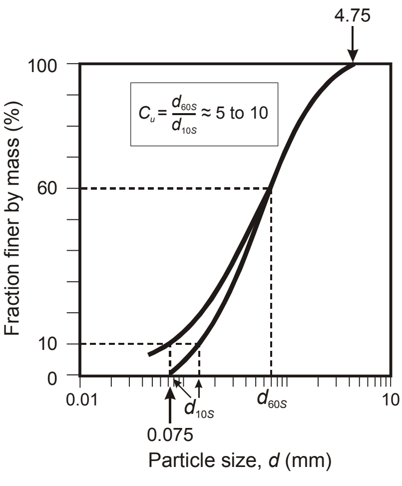

Figure 30. Range of particle size distributions of soil with zero to 10% fines and a maximum particle size of 4.75 mm.

Influence of compressive stress on opening size

Needle-punched nonwoven geotextiles are compressible. The porosity of a needle-punched nonwoven geotextile, which is linked to the thickness through Equation 36, decreases as the thickness decreases under compressive stress. Approximate relationships, based on some experimental data, between porosity and compressive stress for typical needle-punched nonwoven geotextiles are presented in Figure 31.

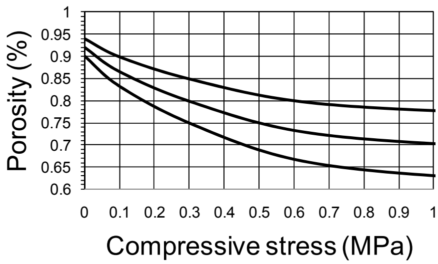

Figure 31. Relationship between porosity of typical needle-punched nonwoven geotextiles and compressive stress for initial porosities of 0.90, 0.92 and 0.94.

Equations 35 and 37 make it possible to calculate the opening size of a needle-punched nonwoven geotextile filter under compressive stress. For example, in Valcros Dam (see Section 7), the overburden stress on the downstream drain is of the order of 0.1-0.15 MPa. Under this stress, the geotextile porosity is about 0.85, as seen in Figure 31 on the curve for a geotextile having an initial porosity of 0.92 (the initial porosity of the geotextile used at Valcros Dam, as indicated in Section 7.1). Using Equation 37 with $n = 0.85$, $\mu_{GT} = 0.3$ kg/m^2, $\rho_f = 1380$ kg/m^3 and $d_f = 0.000027$ mm gives 0.071 mm. Comparing this result with the value of 0.099 mm obtained under no stress (see Section 7.4), it appears that the opening size reduction under a relatively small compressive stress is significant.

It is useful to be able to calculate the opening size of a needle-punched nonwoven geotextile filter, because the results of tests conducted to measure opening sizes are often inaccurate. The calculation is reliable because the equations have been calibrated experimentally, and because the parameters of the equations (geotextile thickness, geotextile mass per unit area, fiber density, and fiber diameter) are determined using reliable physical tests and the porosity is calculated from three of these parameters using a straightforward equation (Equation 36).

It is useful to be able to calculate the opening size of a needle-punched nonwoven geotextile under stress, because compressive stresses have a significant impact on the opening size, as shown above, and because there is no test to do this measurement.

* * * * *