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Reinforced clay - A preliminary study using the triaxial apparatus

Argile armée - Etude préliminaire à l'appareil triaxial

Cette communication présente les résultats d'une étude préliminaire de la rupture d'éprouvettes cylindriques d'argile armée.
Deux types d'armatures ont été utilisées avec des écarts différents.

INTRODUCTION

Reinforced earth may be considered to comprise of two main components, the reinforcement and the soil backfill. Simple criteria issued by the Societe La Terre Armée require that the backfill should contain not more than 15% by weight, of material passing the 80 µm sieve, Schlosser (1976). This specification therefore precludes the use of clay. The use of cohesive soil in reinforced earth poses considerable problems. It has been pointed out that even if the shear strength of a cohesive soil is fully mobilised the maximum possible bond stress that could develop would be equal to the undrained shear strength of the soil. This would be low compared to the bond stress developed by the granular backfill normally used and might even involve a change in the reinforced earth technique, Schlosser and Vidal (1969). The small amount of test data available for reinforced clay gives pessimistic results. An investigation into the reinforcing effect of straw on unfired clay bricks by Razani and Behpour (1970) concluded that the reinforcement increased ductility but actually decreased compressive strength. The decrease in strength was quite dramatic with the addition of 2.5% by weight, of straw halving the compressive strength. A limited study has been reported by Lee (1976) who carried out unconfined compression tests on the samples of compacted clay reinforced with thin narrow strips of mylar tape. The results showed an increase in the ductility of the sample but no increase in compressive strength. Despite these somewhat depressing findings it was decided to carry out a pilot study of bond failure of reinforced clay using the triaxial apparatus. Before this, however, a crude analytical assessment was made.

THEORY

It was considered that classical forging theory might have some application to reinforced earth, Butterfield (1977). This theory which considers the compression of a thick disc of material undergoing compression between frictional platens is set out briefly.

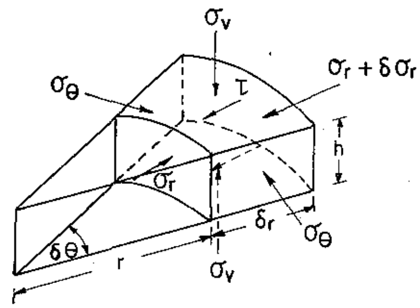


Figure 1 STRESSES ON CYLINDRICAL ELEMENT

Consider the radial equilibrium of a disc of material of radius R and thickness h being compressed between platens. It follows from Figure 1 that -

$$\sigma_r\, r\, h\, \delta\theta + 2\, \sigma_\theta\, h\, \delta r\, \frac{\delta\theta}{2} = 2\tau\, r\, \delta r\, \delta\theta + h\,(\sigma_r + \delta\sigma_r)\,(r+\delta r)\,\delta\theta$$

Ignoring second order and higher terms this expression simplifies to

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = -\frac{2\tau}{h} \quad \ldots\ldots\ldots\ldots\ldots\ldots 1$$

It may be assumed, for small values of τ, that the radial and circumferential stress components σ_r and σ_θ are equal and that both are principal stresses, Siebel (1923). On this basis equation 1 becomes -

$$2\tau\, dr + h\, d\sigma_r = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots 2$$

Equation 2 may be adapted for application to reinforced earth. Consider first a dry purely frictional soil obeying the Mohr - Coulomb failure criterion which may be expressed in the form -

$$(\sigma_1 - \sigma_3) = (\sigma_1' + \sigma_3')\, \sin\phi \quad \ldots\ldots\ldots\ldots\ldots\ldots 3$$

Assuming that the angle of bond stress is δ then the shear stress at soil/reinforcement interface may be expressed as -

$$\tau = \sigma'_v \tan\delta \quad \ldots\ldots\ldots\ldots\ldots\ldots 4$$

If δ is the peak value of the angle of bond stress then this value will only be mobilised if there is sufficient relative strain between the soil and reinforcement. As a first approximation it is assumed that relative radial strain and hence the proportion of tan δ mobilised is a linear function of the radial distance from the centre of the sample where the strain is assumed zero.

These assumptions give -

$$\tau = \frac{r}{R} \tan \delta \; \sigma v' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 5$$

For small values of τ it may be further assumed that the vertical effective stress is principal whence equation 5 becomes -

$$\tau = \frac{r}{R} \tan \delta \; \sigma_1' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 6$$

Similarly $d\sigma r = Kad\sigma_1'$ Substitution into equation 2 then leads to equation 7 -

$$\frac{2\tan\delta}{R} \sigma_1' \, r \, d\sigma r + Kahd\sigma_1' = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots 7$$

On separation of variables this becomes -

$$\frac{d\sigma_1'}{\sigma_1'} + \frac{2 \tan\delta}{hKaR} \, rdr = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 8$$

On integration of equation 8 an expression is obtained for σ_1' -

$$\sigma_1' \; Ce^{-r^2\tan\delta/KahR} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 9$$

Equation 9 may be re-written in terms of the minor principal effective stress -

$$\sigma_3' = C \; Ka \; e^{-r^2\tan\delta/KahR} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 10$$

Considering now the boundary conditions, when r=R the principal stress equals the applied confining pressure which is denoted by σ_c'.

$$\sigma c' = C \; Ka \; e^{-R\tan\delta/Kah} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 11$$

Whence -

$$C = \sigma c' Kp \; e^{R\tan \delta/Kah}$$

Substitution for the constant in equation 10 gives -

$$\frac{\sigma_3'}{\sigma_c'} = e^{\tan\delta(R^2-r^2)/KahR} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 12$$

The average value of equation 12 is given by integration -

$$\left(\frac{\sigma_3'}{\sigma_c'}\right)_{ave} = \frac{1}{R}\int_0^R e^{\tan\delta(R^2-r^2)/KahR} \, dr \ldots 13$$

Equation 12 cannot be integrated directly, however, it was found by numerical methods that the antilog of the average value of $\log_e (\sigma_3'/\sigma_c')$ was, to a close approximation, equal to the average value of (σ_3'/σ_c') thus -

$$\log_e \left(\frac{\sigma_3'}{\sigma c'}\right)_{ave} = \frac{\tan \delta}{KahR^2}\int_0^R (R^2-r^2) dr \ldots 14$$

Integration and transformation of equation 14 gives -

$$\left(\frac{\sigma_3'}{\sigma_c'}\right)_{ave} = e^{2 \tan \delta R/3Kah} \quad \ldots\ldots\ldots\ldots\ldots\ldots 15$$

Equation 15 may be expressed in a more meaningful manner by considering the increase in minor principal stress. $\Delta\sigma_3'$ caused by the reinforcement. Reverting to the use of σ_3' to denote confining pressure.

$$\frac{\sigma_3' + \Delta\sigma_3'}{\sigma_3'} = e^{\tan \delta/3Kah\alpha} \quad \ldots\ldots\ldots\ldots 16$$

Where α is the aspect ratio, h/2R, for each element of a reinforced cylindrical sample.

It is useful to compare the predictions made using equation 16 with test data published by Yang(1972) who carried out triaxial tests on samples of sand, of different aspect ratios reinforced with rigid steel reinforcing discs. The published value of Kp for the sand renders a ϕ value of 39°, a value for δ is derived using the δ/ϕ value of 0.54 published by Potyondy (1961). This is in good agreement with values measured by the Author and those reported by Schlosser and Vidal (1969). Using these values equation 16 leads to the theoretical line given in Figure 2. As can be seen there is good agreement between the test results and the theoretical line for aspect ratio less than unity.

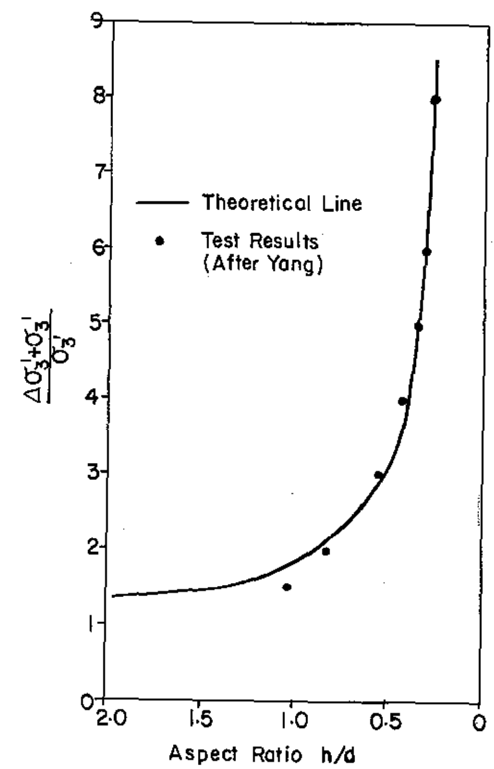


Figure 2 STRENGTH RATIO vs ASPECT RATIO

The proceeding theory has been stated in terms of effective stress analysis which is of course only applicable to the long term analyses of clay soils. Since the short term condition is likely to be more critical it is necessary to revert to equation 2 and consider a total stress analysis. However before doing this it is worthwhile considering an intermediate condition whereby use of an appropriate reinforcement complete drainage can be effected at the soil/reinforcement interface.

It is taken that the failure criterion for the clay can be expressed in the form of equation 17.

$$2Cu = (\sigma_1 - \sigma_3) \qquad \text{.....................17}$$

From equation 17 it follows that at failure $d\sigma_1 = d\sigma_3$. Since $d\sigma_r$, equation 2, is assumed to be a principal stress it further follows that $d\sigma_1 = d\sigma_3 = d\sigma_r$. This leads from equation 2 to equation 18.

$$2\tau dr + h d\sigma_1 = 0 \qquad \text{.....................18}$$

Considering unconfined compression and reiterating the assumptions made in leading to equation 6 -

$$\frac{2\tan\delta}{R} \sigma_1 r dr + h d\sigma_1 = 0 \qquad \text{.....................19}$$

Separating variables and integrating gives equation 20

$$\sigma_1 = C \; e^{-r^2 \tan\delta / hR} \qquad \text{.....................20}$$

When $r = R$, $\sigma_3 = 0$ thus $\sigma_1 = 2Cu$, whence -

$$C = 2Cue^{R\tan\delta/h} \qquad \text{.....................21}$$

Combining equations 20 and 21 -

$$\frac{\sigma_1}{2Cu} = e^{\tan\delta(R^2 - r^2)/hR} \qquad \text{.....................22}$$

The average value of equation 22 is -

$$\left(\frac{\sigma_1}{2Cu}\right)_{ave} = e^{2\tan\delta \; R/3h} \qquad \text{.....................23}$$

For a sample subjected to confined compression -

$$\frac{(\sigma_1 - \sigma_3)r}{2Cu} = e^{\tan\delta/3\alpha} \qquad \text{.....................24}$$

Where, as before, α is the aspect ratio $h/2R$. The suffix r denotes reinforced.

Reverting to equation 16 it may be assumed that if the clay, in the drained condition, obeys the Mohr - Coulomb criterion, that $\Delta\sigma_3'$ induces a corresponding increase in σ_1' of $Kp\Delta\sigma_3'$. Thus the drained strength of the reinforced sample could be expressed as a ratio of the drained strength of the unreinforced sample by equation 25.

$$\text{Strength ratio} = \frac{(\sigma_1' + \Delta\sigma_1') - (\sigma_3' + \Delta\sigma_3')}{(\sigma_1' - \sigma_3')} \qquad \text{.................25}$$

This reduces to -

$$\text{Strength Ratio} = \frac{Kp(\sigma_3' + \Delta\sigma_3') - (\sigma_3' + \Delta\sigma_3')}{(Kp \; \sigma_3' - \sigma_3')} \qquad \text{.... 25a}$$

Further simplification gives :

$$\text{Strength Ratio} = \frac{\sigma_3' + \Delta\sigma_3'}{\sigma_3'}$$

From this it follows that for the fully drained condition

$$\frac{(\sigma_1' - \sigma_3')r}{(\sigma_1' - \sigma_3')u} = e^{\tan\delta/3Ka\alpha} \qquad \text{.....................26}$$

Where the suffixes r and u denote reinforced and unreinforced respectively. If $\phi u = 0$ analysis is applied to equation 26 it is found that equation 24 is obtained.

Equation 24 can only be valid if the shear stress transmitted to the clay between consecutive reinforcements does not exceed the undrained shear strength of the clay. Shear failure of the clay mass can be assessed by the appropriate adaption of equation 2. As a lower bound solution it is taken that drainage of the clay by the reinforcement does not extend beyond the clay/reinforcement interface. Using the previously made assumptions equation 2 becomes -

$$\frac{2\tan\delta}{R} \sigma_1 r \; dr + h d\sigma_1 = 0 \qquad \text{................27}$$

Integration of equation 27 and use of the boundary conditions $r = R$, $0_1 = 2Cu$ leads to equation 28.

$$\frac{(\sigma_1 - \sigma_3)r}{2Cu} = 1 + 1/4\alpha \qquad \text{.....................28}$$

The above expression would be equally applicable to a solid reinforcement where the soil/reinforcement bond stress would be a function of the undrained shear strength of the clay. If this adhesion is taken to be μCu $(\mu < 1)$, then equation 28 becomes -

$$\frac{(\sigma_1 - \sigma_3)_r}{2Cu} = 1 + \mu/4\alpha \qquad \text{.....................29}$$

From the foregoing it is possible to define three possible strength ratios for samples failing in bond -

i. Fully drained : - $e^{\tan\delta/3Ka\alpha}$

ii. Full drainage soil/reinforcement interface only. : $e^{\tan\delta/3\alpha}$

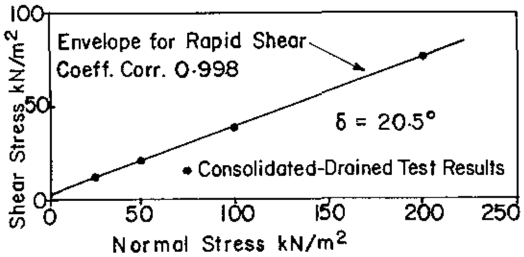
iii. Totally undrained $1 + \mu/4\alpha$

In (ii) above it is possible for internal failure to occur before bond failure. This is a special case of (iii) above.

iv. Internal failure : - $1 + 1/4\alpha$

TESTS AND TESTS RESULTS

Three series of unconfined compression tests were carried out. In the first two series 102 mm diameter by 204 mm high samples of Kaolin clay were used. These were reinforced with 102 mm diameter porous discs with two samples being made for each reinforcement spacing. To check the effects of soil/reinforcement stiffness batches of samples with nominal undrained strengths of 50kN/m^2 and 150 kN/m^2 were used. Earlier shear box tests, Figure 3, showed that this material drained so rapidly that both fully drained tests and rapid tests gave the same angle of bond stress of 20.5°. In the first series of tests the samples



Figre 3. SHEAR BOX TEST RESULTS

TABLE 1
RESULTS OF RAPID SHEAR TESTS ON 102mm. DIAMETER SAMPLES

No.OF DISCS	$\frac{(\sigma_1-\sigma_3)}{2}$ kN/m^2	MOISTURE CONTENT %	C_u kN/m^2	$\frac{(\sigma_1-\sigma_3)}{2C_u}$ kN/m^2	AVERAGE	DISC SPACING mm.	$\frac{h}{d}$
4	30	33·10	47·40	0·63	0·79	68·30	0·62
4	54	32·10	56·80	0·95			
6	50	33·20	46·60	1·07	0·94	40·90	0·35
6	44	32·30	54·80	0·80			
9	113	30·60	75·20	1·50	1·70	25·60	0·20
9	95	32·80	50·00	1·90			
12	305	30·80	72·40	4·21	3·92	18·60	0·14
12	253	31·00	69·70	3·63			
3	79	28·60	111·20	0·71	0·71	106·80	1·00
3	78	28·70	109·70	0·71			
4	128	27·00	157·00	0·82	0·79	68·30	0·62
4	123	26·80	164·00	0·75			
6	193	28·30	119·10	1·62	1·30	40·90	0·35
6	140	27·40	144·00	0·97			
9	185	27·80	132·20	1·40	1·45	25·60	0·20
9	263	26·50	175·20	1·50			

Notes:- i) h is height of soil element between consecutive reinforcing discs.
ii) d is sample diameter.

were sheared rapidly at an overall rate of 2%/min. The results are given in Table 1. The undrained shear strength of the clay in each sample was determined by measuring its moisture content. Earlier tests on unreinforced samples showed that a log-log linear relationship existed between undrained shear strength and moisture content, Figure 4.

In the second series of tests the reinforced samples were sheared slowly thus allowing full drainage. The results for these tests are given in Table 2.

The strength of the unreinforced soil was taken to be equal to half the deviator stress at failure of samples with an aspect ratio of two. In the third series of tests 38 mm x 76 mm samples of Kaolin were reinforced with 0.02 mm thick aluminium foil and sheared rapidly, the results are given in Table 3.

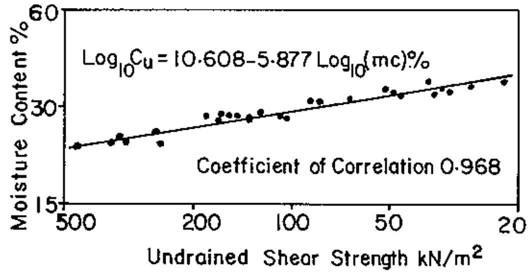


Figure 4. UNDRAINED SHEAR STRENGTH/ MOISTURE CONTENT

TABLE 2
RESULTS OF DRAINED SHEAR TESTS ON 102 mm DIAMETER SAMPLES

No OF DISCS	DEVIATOR STRESS kN/m^2	MOISTURE CONTENT %	AVERAGE DEVIATOR STRESS kN/m^2	STRENGTH RATIO	DISC SPACING mm	$\frac{h}{d}$
2	55·20	32·90	63·10	1·00	204·80	1·96
2	71·00	30·30				
4	69·40	30·80	99·30	1·57	68·30	0·62
4	129·20	28·30				
6	115·80	29·80	110·90	1·76	40·90	0·35
6	106·00	31·40				
9	178·60	31·30	180·00	2·85	25·60	0·20
9	181·40	30·00				
12	432·20	31·90	443·60	7·03	18·60	0·14
12	455·00	30·60				
2	158·00	29·50	165·80	1·00	204·80	1·96
2	173·60	29·20				
4	213·40	26·90	218·70	1·32	68·30	0·62
4	224·00	26·80				
6	258·00	27·40	241·50	1·46	40·90	0·35
6	225·00	29·20				
9	404·00	27·70	411·00	2·48	25·60	0·20
9	418·00	23·80				
2	88·60	33·50	98·20	1·00	204·80	1·96
2	107·80	32·50				
3	112·40	33·50	121·40	1·24	106·80	1·00
3	130·40	32·30				

TABLE 3
RESULTS OF RAPID SHEAR TESTS ON 38 mm DIAMETER SAMPLES

No OF DISCS	$\frac{(\sigma_1-\sigma_3)}{2}$ kN/m²	MOISTURE CONTENT %	Cu kN/m²	$\frac{(\sigma_1-\sigma_3)}{2Cu}$	AVERAGE	DISC SPACING mm	$\frac{h}{d}$
4 4 4	16 135 275	33·00 26·00 14·00	48·30 196·00 505·0*	0·33 0·69 0·54	0·52	25·30	0·67
5 5 5	16 76 282	35·00 26·40 14·00	34·20 179·20 505·0*	0·46 0·42 0·56	0·48	19·00	0·50
7 7	13 87	34·30 26·90	38·50 160·50	0·34 0·54	0·44	12·70	0·33

Notes – * Undrained shear strength determined by direct measurement.

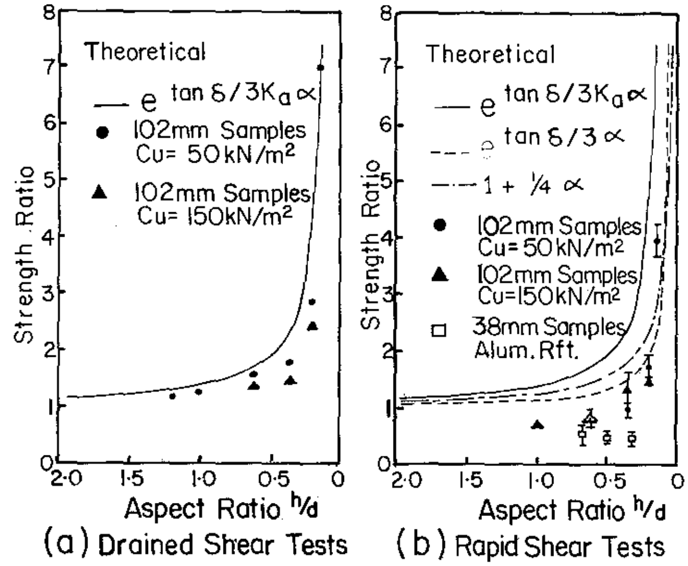


Figure 5. STRENGTH vs ASPECT RATIO

Figure 5a shows the results for the fully drained tests together with the theoretical line. The results confirm the general validity of the theory in as much as the strength ratio increases exponentially with decreasing aspect ratio. The tests results fall below the theoretical line, this is most probably due to the fact that the reinforcement is not, as assumed in the theory, infinitely stiff compared to the soil. The effect of this is to reduce the relative radial strain between soil and reinforcement and therefore reduce the magnitude of the bond stress mobilised. This seems to be borne out by the fact that the strength ratios for the stiffer clay with a strength of 150 kN/m² are less than those for the clay with a strength of 50 kN/m².

At low aspect ratios, the results for the rapid shear tests, Figure 5b, again show an exponential increase in the strength ratio. The lowest aspect ratio of 0.14 gives a strength ratio that falls between the theoretical lines, this is reasonable since with the porous discs at such close centres it would be expected that some drainage would occur even at rapid rates of shear. At higher aspect ratios the results fall closer to the line predicted using the assumption that drainage is restricted to the soil reinforcement interface. The most surprising results are those that fall between aspect ratios of 0.5 and unity where the addition of reinforcement appears to give a 20% to 30% reduction in the strength of the reinforced clay. Results from the tests on 38 mm diameter samples reinforced with aluminium showed on average a 50% decrease in strength for aspect ratios between 0.33 and 0.67.

The modes of failure causing these strength reductions fall into one of two categories. In the case of the foil reinforced samples it was found on dismantling the samples that the clay exhibited both radial and circumferential tension cracks. It is thought that there was a low bond stress generated between the clay and the aluminium thus allowing the development of radial tensile stresses leading to premature failure.

The particular test results indicated a strength decrease of 50%. It is interesting to note that Berg(1950) reported an identical decrease in strength in concrete samples compressed using lubricated end platens as opposed to conventional platens, Berg observed microfissures develop during testing and concluded that lubricated end platens permitted the development of horizontal tensile forces which resulted in premature failure.

The second possible mode of failure can be expressed in more quantitative terms by reference to the results from two consolidated undrained multi-stage triaxial compression tests with pore water pressure measurement, Table 4. Both samples which were 102 mm x 204 mm, were made from remoulded London Clay. The first sample, which served as a control sample was tested at cell pressures of 100 kN/m², 200 kN/m² and 400 kN/m². The second sample which was tested at the same cell pressures was reinforced with 102 mm diameter discs of 0.02 mm thick aluminium foil at 25 mm centres. The samples were sheared at a rate of strain consistent with complete equalisation of the porewater pressures generated.

Examination of Table 4 shows what appear to be extremely high values of the porewater pressure coefficient A. These values have been calculated in the normal manner for a fully saturated soil.

$$\Delta u = A\ (\sigma_1 - \sigma_3)$$

TABLE 4
RESULTS OF MULTISTAGE TRIAXIAL TESTS

SAMPLE	CELL PRESSURE σ_3 kN/m²	$\sigma_1-\sigma_3$ kN/m²	Δu kN/m²	A	$\Delta\sigma_3$ kN/m²	STRENGTH RATIO
Control	100 200 400	52 108 191	50 101 201	0·96 0·94 1·05	— — —	1·00 1·00 1·00
Reinforced	100 200 400	32 82 151	71 126 264	2·22 1·54 1·75	21 25 63	0·62 0·76 0·79

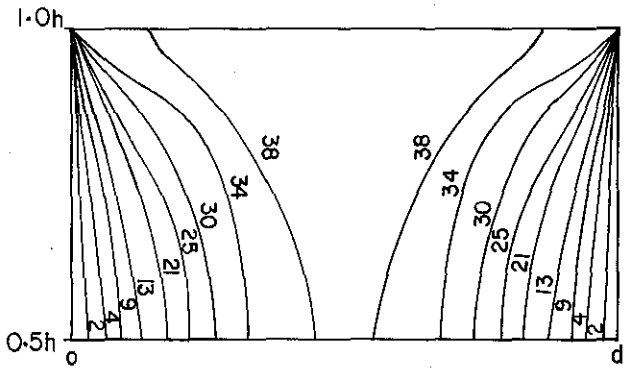


Figure 6. DISTRIBUTION OF $\Delta\sigma_3$

If however it is accepted that the coefficient A is sensibly a constant of the soil for these two tests then this leads to the conclusion that there is an increase in confining pressure $\Delta\sigma_3$ induced in the reinforced sample. From the value of u measured for the reinforced samples and A values for the unreinforced samples the magnitude of $\Delta\sigma_3$ is -

$$\Delta\sigma_3 = \Delta u - A(\sigma_1 - \sigma_3)$$

Since the samples were saturated it would be expected that a uniform increase in total stress $\Delta\sigma_3$ would induce a uniform increase in porewater pressure equal in magnitude to $\Delta\sigma_3$. This of course would cause no change in effective stress and therefore not influence the behaviour of the sample. However finite element analyses, indicate that the distribution of $\Delta\sigma_3$ may be far from uniform.

Figure 6 shows the results of an analysis on an unconfined samples with an aspect ratio of 0.29, Yang(1972). The figures show the magnitude of $\Delta\sigma_3$ as a percentage of the applied axial stress. It is postulated that high values of $\Delta\sigma_3$ generated near the centre of the sample induce correspondingly high porewater pressures which are transmitted through the sample to areas of low $\Delta\sigma_3$.

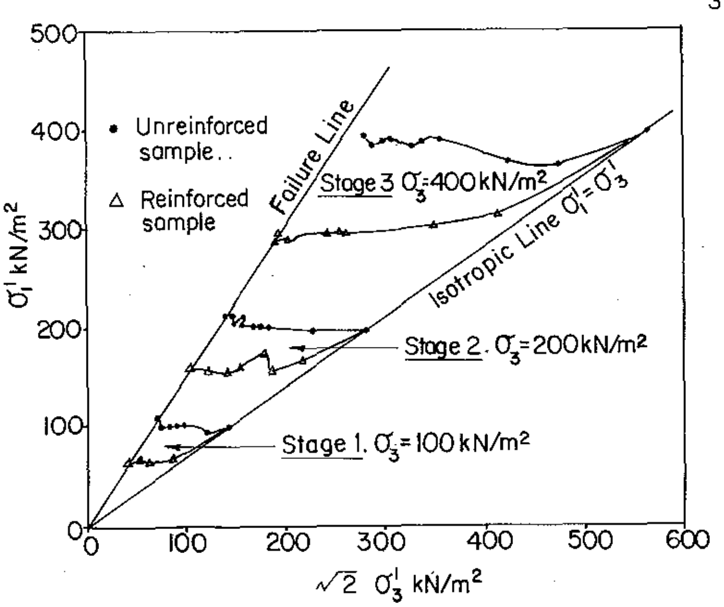


Figure 7. STRESS PATH DIAGRAM

These high transmitted porewater pressures would then cause a reduction in effective stress and hence induce a premature failure. Such a mode of failure is indicated by the stress plot in Figure 7. This shows that at low initial strains there is a decrease in effective confining pressure leading to early failure.

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