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Reinforcement force in embankments on soft soils

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ABSTRACT: The approach suggested by Low for the analysis of the rotational stability of earth embankments has been extended to reinforced embankments on soft soils. The results obtained for a few cases of reinforced embankments using the new solution, the Low et al's solution, and other solutions have been compared.

INTRODUCTION

It is now well recognised that a reinforcement introduced in a sheet or grid form at the base of the embankment built on soft soil improves its rotational stability. Commonly the limit equilibrium approach is used in the analysis of both the unreinforced and reinforced embankments.

UNREINFORCED EMBANKMENTS

For the embankment, without the reinforcement, shown in Fig.1 Low (1989) has shown that the critical slip circle tangential to a given limiting tangent is a mid-point circle and the factor of safety F_o for this critical circle is given by

$$F_o = N_1 \frac{c_a}{\gamma H} + N_2\left(\frac{c}{\gamma H} + \lambda \tan\phi\right) \qquad (1)$$

where c_a is the average undrained shear strength of the foundation soil within the depth D to the limiting tangent. The other parameters are shown in the figure. Stability factors N_1, N_2, and the coefficient λ are functions of D/H and n. These are given by Low in the form of charts and equations. The overall minimum factor of safety can be obtained by considering different limiting tangents.

REINFORCED EMBANKMENTS

Solution based on Low's analysis

In the reinforced embankment shown in Fig.1 the reinforcement is placed at a above the ground surface. The origin of axes X and Y is at the intersection of the vertical line through the toe and the limiting tangent. For an arbitrary slip circle with centre at (X_o, Y_o) the factor of safety of the reinforced embankment F is defined as

$$F = \frac{M_{RT}}{M_o} = \frac{M_{RU} + M_{RR}}{M_o} \qquad (2)$$

where M_{RT} is the total restoring moment which consists of moment M_{RU} due to shear stresses along IGEJ and moment M_{RR} due to the reinforcement force. M_o is the overturning moment. Low (1989) gives the expressions for M_{RU} and M_o. Assuming the reinforcement force P at point K to act horizontally, M_{RR} is written as

$$M_{RR} = P\left(Y_o - D - a\right) \qquad (3)$$

Substituting for M_{RR} from Eq.3 in Eq.2 and rearranging the terms the reinforcement force is expressed as

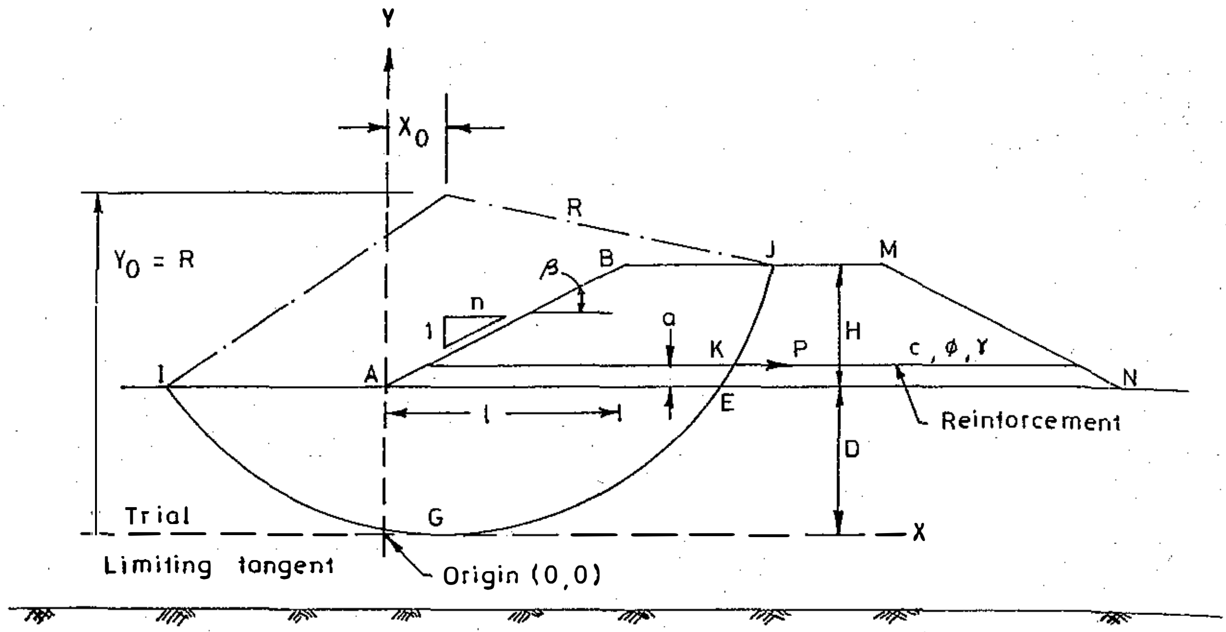

Fig.1 Reinforced embankment on soft soil

$$P = \frac{M_o F - M_{RU}}{Y_o - D - a} \tag{4}$$

Two equations are now set up by considering the partial derivatives of P with respect to X_o and Y_o and equating to zero. The location of the centre of the slip circle for maximum reinforcement force is obtained by solving the two equations. The solution shows that the circle is a mid-point circle and Y_o is obtained by solving the Eq.5.

$$\left(\frac{Y_o}{H}\right)^{1.47} - 3.128\left(\frac{D+a}{H}\right)\left(\frac{Y_o}{H}\right)^{0.47} - 2.128\left(\frac{F_2}{F_1}\right) = 0 \tag{5}$$

where F_1 and F_2 are given by separate equations (Kaniraj and Abdullah 1992). Solutions for Y_o/H are given in Fig.2.

Substituting these X_o and Y_o in Eq.4, the expression for maximum reinforcement force P_{max} for a given limiting tangent is written as

$$P_{max} = \gamma H^2 (A_1 F - D_1 F_1) \tag{6}$$

where A_1, D_1 and F_1 are given by separate equations. The maximum required reinforcement force can be determined by considering different limiting tangents.

Proceeding in the same way as explained above, solutions can be obtained if the reinforcement force is assumed to act tangentially to the failure surface. In this case the following changes are to be incorporated in Eqs 3 to 6:

Equation 3, for M_{RR}, is written as

$$M_{RR} = P\, Y_o \tag{7}$$

Equation 4, for P, is written as

$$P = \frac{M_o F - M_{RU}}{Y_o} \tag{8}$$

Y_o is obtained from Eq.5 with the middle term as zero. Solutions for Y_o/H are presented in Fig.3. P_{max} is given by Eq.6 with A_1 and D_1 now given by different equations.

Low et al's solution

Low et al (1989) consider that the reinforcement force reduces the overturning moment and define

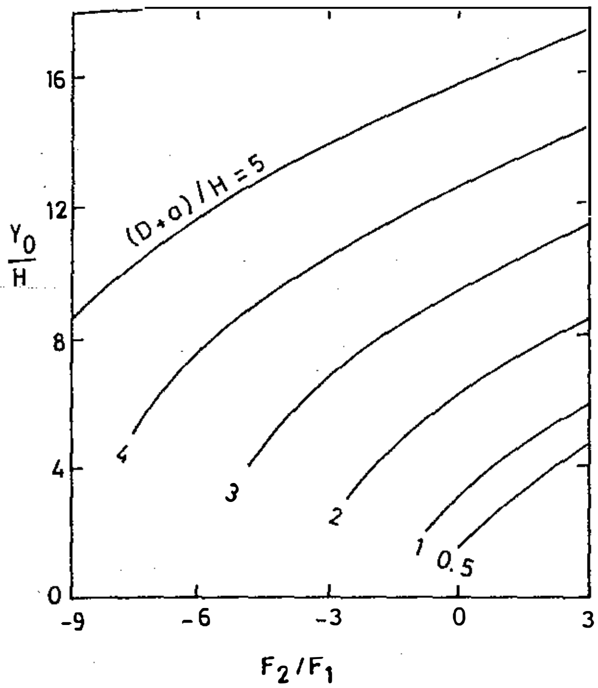

Fig.2 Y_o/H for horizontal force

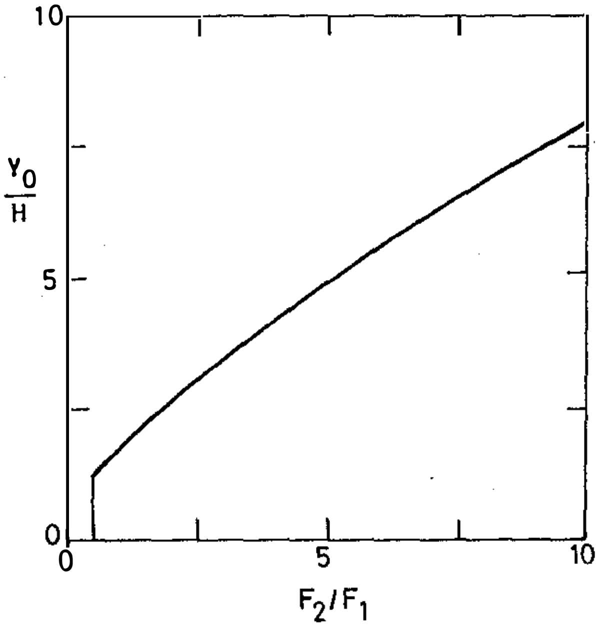

Fig.3 Y_o/H for tangential force

the factor of safety of the reinforced embankment F_L as

$$F_L = \frac{M_{RU}}{M_o - M'_{RR}} \quad (9)$$

where M_{RR}' is the restoring moment due to the reinforcement force T. Assuming T to act horizontally, M_{RR}' is written as

$$M'_{RR} = T(Y_o - D - a) \quad (10)$$

T is given by

$$\frac{T}{\gamma H^2} = \frac{1 - \frac{F_o}{F_L}}{I_R} \quad (11)$$

where F_o is the minimum factor of safety of the unreinforced embankment. I_R is defined as a stability number dependent on D/H and n, and is given in the form of a chart.

Comparison of the two solutions

The definitions adopted for the factor of safety of reinforced embankments in Eqs 2 and 9 are different though Eq.2 is the conventional one. Therefore, for the same numerical value of target factor of safety $F = F_L$ the reinforcement forces obtained by the two solutions would be different. For Eqs 2 and 9 to give the same factor of safety it can be shown that

$$T = \frac{P_{max}}{F} \quad (12)$$

Thus, while the solution presented in the paper gives the maximum reinforcement force for the critical slip circle, Low et al's solution gives the working reinforcement force.

In the present solution the location of the critical circle and the maximum reinforcement force are given in the form of closed form equations. These can be, therefore, precisely calculated. In the case of Low et al's solution, approximation and interpolation of values are required in the use of the stability number chart.

Low et al have presented the results for the following four cases:

Case 1: H = 6 m, c = 0, $\phi = 30°$, $\gamma = 20$ kN/m^3, n = 2, H_s = 4 m. The foundation soil

Table 1. Comparison of the results for Cases 1 to 3

	New	Low et al
Case 1: D = 4 m, F = 1.3		
T (kN/m)	236.8	237
Case 2: D = 3 M, F = 1.35		
Y_o/H	2.21	2.22
T (kN/m)	137.3	141
Case 3: D = 3 M, F = 1.35		
Y_o/H	2.14	2.14
T (kN/m)	60	62

Table 2. Comparison of the results for T (kN/m) for Case 4 (F = 1.3)

D (m)	New	Low et al
3.0	108.5	108
4.5	165.5	163
6.0	197.3	189
7.5	195.0	189
9.0	153.2	147
10.5	74.1	74

has a uniform undrained cohesion of 18 kPa

Case 2: H = 6 m, c = 20 kPa, $\phi = 0°$, $\gamma = 19.4$ kN/m³, $\beta = 30°$, $H_s = 3$ m. The foundation soil has a uniform undrained cohesion of 20 kPa

Case 3: H = 6 m, c = 0, $\phi = 37°$, $\gamma = 17$ kN/m³, $\beta = 30°$, $H_s = 3$ m. The foundation soil has a uniform undrained cohesion of 20 kPa

Case 4: H = 4 m, c = 0, $\phi = 30°$, $\gamma = 19$ kN/m³, n = 2, $H_s = 12$ m. The foundation soil has a bilinear undrained strength profile. The undrained cohesion decreases from 15 kPa at the

Table 3. Results for Case 1 using new solutions and EMSOFGM program $P_{max} = 308$ kN/m

Direction	D (m)	Y_o (m)	F
Horizontal	3.11	14.94 (14.93*)	1.421
	3.29	14.92	1.393
	3.47	14.93	1.367
	3.64	14.94	1.344 (1.343)
	3.82	14.97 (14.96)	1.321
	4.00	15.00	1.300
Tangential	3.11	18.48	1.527
	3.29	18.63	1.501
	3.47	18.76 (18.79)	1.478
	3.64	18.95 (18.94)	1.457 (1.456)
	3.82	19.11	1.436
	4.00	19.29	1.417

*Figures within brackets are the values from EMSOFGM program where they are different from the new solutions

ground surface to 10 kPa at 3 m below the ground surface and then increases to 25 kPa at 12 m below the ground surface.

Values of T and Y_o/H calculated for the four cases using the new solution are presented along with Low et al's values in Tables 1 and 2.

Comparison with other solutions

Cases 1 and 4 have been analysed for P_{max} equal to 308 kN/m and 260 kN/m, respectively, by Low et al using a computer program named EMSOFGM. Almost identical results were obtained using the new solution also. Table 3 and 4 give the results for Y_o and F obtained by using the new solution and the EMSOFGM

Table 4. Results for Case 4 using new solutions and EMSOFGM program P_{max} = 260 kN/m

Direction	D (m)	Y_o (m)	F
Horizontal	2.55	10.46 (10.47*)	1.772 (1.773)
	3.49	10.18	1.515
	4.44	10.33	1.386 (1.387)
	5.38	10.87	1.323
	6.33	11.73	1.300
	7.27	12.81	1.303
	8.22	14.05	1.323
	9.16	15.39	1.355
	10.11	16.79 (16.80)	1.396
	11.05	18.24	1.442
	12.00	19.72 (19.73)	1.494
Tangential	2.55	15.23 (15.22)	1.970
	3.49	16.35	1.737
	4.44	17.38	1.624
	5.38	18.41	1.565
	6.33	19.47	1.539
	7.27	20.56	1.535
	8.22	21.70	1.547
	9.16	22.85	1.569
	10.11	24.05 (24.04)	1.600
	11.05	25.25	1.637
	12.00	26.49	1.679

*Figures within brackets are the values from EMSOFGM program where they are different from the new solutions

program.

Huisman (1987) presents the results for an embankment given in Case 5.

Case 5: H = 4 m, c = 0, ϕ = 30°, γ = 20 kN/m^3, n = 2, H_s = 4 m. The foundation soil has uniform undrained cohesion of 12 kPa.

Table 5. Results for Case 5 (D = 4 m, F = 1.3)

	New	Huisman
Horizontal reinforcement force		
P_{max} (kN/m)	219.83	224.37
Y_o (m)	10.37	9.3
Tangential reinforcement force		
P_{max} (kN/m)	149.1	140.22
Y_o (m)	14.61	12.1

Table 5 shows the results for P_{max} and Y_o obtained by using the new solution and given by Huisman for this case.

DISCUSSIONS

From the results reported in Tables 1 to 5 it is evident that the new solutions give nearly the same values as those obtained by the other solutions. The new solution and the EMSOFGM program give almost identical results. Low et al's values are within 5% of the values obtained by using the new solutions.

Huisman's solution is approximate. 9 grid points in a square array have been chosen in the analysis. For horizontal reinforcement force, the centre of the slip circle is one of the boundary grid points. For tangential reinforcement force, the centre of the slip circle is the central grid point. Further refinement of its location is possible.

CONCLUSIONS

A new solution for the analysis of the rotational stability of reinforced embankments on soft soils is presented. The approach is based on the method of analysis proposed by Low for unreinforced embankments. The new solution is presented in the form of closed form equations. Low et al too have developed an approach for reinforced embankments. Their solution was presented in the form of a stability number chart. Comparison of the results for example problems obtained by the two solutions and an EMSOFGM computer program shows that the new solutions give almost the same values as those obtained by the other solutions.

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