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Resistance to Area Change as a Measure of Fabric Performance

La résistance des tissus au changement de superficiel comme mesure de performance

The paper is concerned with the resistance of fabrics to area change, a parameter with at least two important effects on the performance of geotextiles. When conformability is required, in order to fit a fabric to a three-dimensional configuration, this can only be accommodated by area change. On the other hand a loss of support occurs when an underlaid fabric grows in area.

The paper includes a discussion of the energy-area strain relations and the results of preliminary experiments conducted at UMIST on area changes in conventional fabrics.

Cet article traite le problem de la resistance des tissus au changement de superficie. Ce parametre influence le performance des Geotextiles au niveau de conformité quand un changement de superficie est avantageux, tandis q'une perte de support est inevitable quand un Geotextile sous - posé s'agrandi.

L'article examine les relation energie/surface/extension et présente les resultats des experiences faite a l'UMIST sur des tissus conventionnels.

INTRODUCTION

Textile engineering and civil engineering, which have come together in the new technology of geotextiles, both derive from ancient useful arts. But whereas civil engineering embraced the science of mechanics at any early date and became the senior branch of engineering, the production of textiles remained as a craft and is only now developing a sound basis of mechanical understanding. Although geotextiles have made rapid progress through empirical cooperation between civil engineers and textile manufacturers, with some use of design calculations modified from other contexts and different materials, further advances will require a proper treatment of the mechanics of the systems involved. This will need the tripartite work of applied mathematicians textile engineers and civil engineers, in the same way that bridge-building and other public works have required a combination of mechanics, metallurgy and materials science, and structural engineering.

The present paper is not a definitive account of a completed piece of research. It is an attempt to stimulate thought about the formulation of ways of characterising and analysing the mechanical behaviour of textile materials, so that the tripartite alliance can function productively. An analogy with soil mechanics may be helpful. At first, the civil engineer regarded soil as inert ballast, to be shovelled by navvies rather than studied seriously, but then it was recognised that the mechanical response of the soil was important in relation to the stability of structures; but it also became clear that the behaviour of granular solids was complicated and that scientific progress could only be made when the right modes of description and analysis had been found. As a result, the subject has developed as a specialised sub-branch of mechanics, and a modern text-book will start with an account of the necessary and specific mathematical formulation and go on to engineering design methods and calculations. Similarly, the fabrics used in geotextile applications cannot be treated solely as commercial products of the textile industry, or the concern of fibre chemists, but must be studied as mechanical systems.

Textile materials differ in almost every possible way from the idealisations introduced in the study of the mechanics of materials. They are inhomogeneous, lacking continuity; anisotropic; non-linear even at small strains; easily deformable, suffering large strains and displacements and often achieving success, not failure, through buckling; and, in the literal mathematical sense, complex mechanically, with deformations combining elasticity and viscosity in a reversible and irreversible time-dependent sequence, further complicated by plasticity and frictional slip. Except in the value of an early sketch of a full work, there is little to be gained by treatments which neglect these complications.

In order to reduce an impossible generality to manageable proportions, the simplifications to be looked for are:

(a) the separation of modes of deformation, so that tensile modes can often be neglected as showing no deformation, under low forces and flexural modes neglected, as showing no resistance, under high forces;

(b) the recognition of symmetries which limit the degree of anisotropy;

(c) the contrary assumptions of constant volume deformation of the fibres themselves, and zero resistance to the reduction of volume in the spaces between fibres.

But perhaps the greatest simplifications comes in the fundamental approach, by recognising that textile fabrics must be treated as two-dimensional forms in their own right, and not as a special case of a three-dimensional continuum. There can be no direct recourse to the mechanics of solids. The fabrics are structures with real finite element dimensions of the same order of size as the material thickness, so that structural analysis, which can be disregarded in elasticity theory because the structure is invisible at the molecular level, becomes significant. The most obvious example of this is that bending of a woven fabric does not occur in parallel layers running from extension through a neutral plane to compression, since the fibres and yarns are interlacing from one side of the fabric to the other.

Nevertheless, except when dealing with high-speed dynamic effects where the wavelength becomes comparable to or less than the structural dimensions, a textile fabric can be treated as a two-dimensional continuum. This must be the starting-point for a rational mechanics of geotextiles.

THE MECHANICS OF SHEET MATERIALS

The definitions

The basic treatment of the mechanics of a two-dimensional continuum has been discussed by Shanahan, Lloyd and Hearle (1) and further commented on by Lloyd (2a). A sheet can suffer six independent modes of deformation, which can be superposed by simple addition to give any more complicated form of deformation at a point in the sheet. These modes of deformation are illustrated in Figure 1. In the plane there are two orthogonal tensile modes and a shear mode. Out of the plane, there are two orthogonal bending modes and a twisting mode. In the most general anisotropic material (or in an isotropic material) the choice of axis is immaterial from the point of view of mechanics, but, if there is some structural symmetry, as for example in an orthogonally woven fabric, then simplifications result when the mechanical axes are chosen to coincide with the structural axes.

With six forms of "strain", and correspondingly six "stresses", there is a formal similarity with the three-dimensional system, with its three tensile modes and three shear modes. The six elements of the stress matrix will relate to the six elements of the strain matrix through the modulus or compliance matrices, which, because of their symmetry, reduce from 36 elements to 21 independent elastic constants. Examples of the reduction in the number of independent constants through symmetry are given in the Table 1.

Table 1 Influence of symmetry

Material	Number of independent constants
No symmetry	21
Orthogonal (rectangular, i.e. different x and y)	13
Orthogonal (square, i.e. equal x and y)	6
in plain isotropic	4
isotropic solid	2

In a more general sense, the constants must be regarded as symbols for non-linear functional relationships, probably most conveniently dealt with by recording the strain energy as a general function of six modes of deformation (plus time and hysteresis).

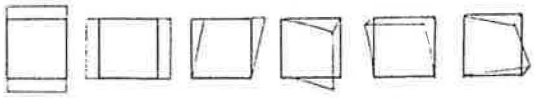

Figure 1. The six independent modes of deformation of sheet material.

Interaction of in-plane and out-of-plane effects

The analogy with the three dimensional case breaks down in one important respect. It is not possible to continue all forms of uniform deformation over finite areas. There is no problem for purely planar deformation, and bending about one axis causes no mathematical problem but does give rise to the physical problem of multiple occupation of space when the material bends back on itself. But twisting of a finite area necessarily involves length changes in the plane; and bending in two directions simultaneously must be accompanied by area changes in the plane.

AREA CHANGE AS A MODE OF DEFORMATION

The analogy with bulk modulus

Giroud (2b) points out that in most geotextile applications the forces are large in comparison with bending resistance, so that it is only necessary to consider resistance to deformation in the plane of the fabric. This simplification will be adopted for the remainder of this paper.

In dealing with bulk materials, although a variety of elastic constants may be used in different circumstances, it is recognised that deformation may be divided into the two categories of dilatational strain and shear strain, giving respectively changes in volume and shape. For isotropic elasticity, a bulk modulus and a shear modulus characterise the material completely. In some applications, shear is dominant and volume changes do not occur; in other applications volume changes are important and the shape alters easily to accommodate these.

In a two-dimensional sheet there will also be shear strain and dilatational strain, with the latter corresponding to area change. The concept of area modulus is not as easy to grasp as that of bulk modulus, because there is no simple experiment corresponding to the change of volume of a solid subject to a hydrostatic pressure, as in figure 2(a). Nevertheless the concept is mechanically identical, as shown in figure 2(b) and (c) with an area change resulting from an equal tensile stress in all directions.

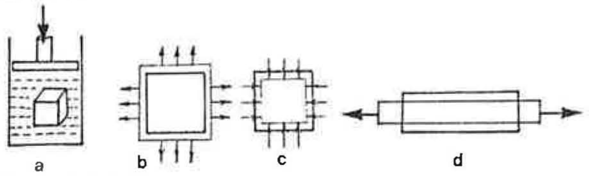

Figure 2 (a) The simple means of measuring volume change
(b) Uniform area extension.
(c) Uniform area compression.
(d) Area change in a solid sheet by change of shape.

In an anisotropic material, the change of volume or area will not occur by means of equal tensile strains in all directions and zero shear, but will involve a change of shape under hydrostatic loading.

Two confusing features

Appreciation of the concept of area change may be blocked by two difficulties.

Firstly in a solid continuum, like a rubber sheet, the area change is achieved by change of shape at substantially constant volume, as shown in figure 2d, and so is related to the shear modulus of the material. However, this does not apply to a structured fabric where in-plane behaviour is not directly linked to thickness change.

Secondly, in the simplest model of a fabric, namely the square pin-jointed assembly of figure 3(a), area increase is strongly resisted, since it can only occur through increase in length of the rods, but area decrease is easy through shearing as in figure 3(b). There is not a discontinuity at the origin, since the area change dA/A associated with shear θ is given by:

$$dA/A = 1 - \cos\theta \approx -\tfrac{1}{2}\theta^2 \qquad (1)$$

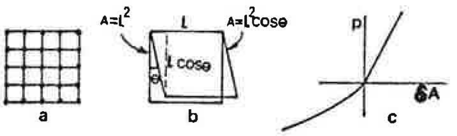

Figure 3. (a) A pin-jointed model of a fabric.
(b) Shear deformation.
(c) Relation between uniform stress p and area change δA.

Shear in either direction leads to area reductions as a second-order quantity, but a quantity which plays an important role in circumstances where second-order effects might be expected to be negligible. The plot of area change against hydrostatic stress will be of the form illustrated in figure 4(c). The initial area modulus is an inadequate description: the reduction in modulus with increasing strain must be considered. We may note, in passing, that figure 4(c) could also represent the pressure-volume relation of a similarly structured solid.

We must also note that the typical statement "shear strains do not cause a change of volume" (3) found in a standard text-book is not necessarily true in a structured material at large strains. In woven fabrics, shear causes area reduction, and the cross-term, the analogue of Poisson's ratio, is important at large strains. Nevertheless although the application of a shear stress may be the easiest way of causing area change, the direct relation shown in figure 3(c) is a simpler way of comparing materials with different forms of anisotropy in terms of the ease with which they can accommodate area change.

Area change is a significant response in geotextile applications in several different ways, which can be recognised as likely to occur in the sixteen applications listed by Giroud (2b).

The significance of area change in geotextile support

Whenever there is any element of support involved, as indicated schematically in figure 5, deformation involves an increase in area of the fabric. If one makes a rough approximation and assumes spherical symmetry and a vertical displacement h spread over an area of radius R, it can be shown that:

$$\text{area strain} = \varepsilon_A = \delta A/A = h^2/R^2, \qquad (2)$$

$$\text{energy of fabric deformation} = \pi R^2 U(h^2/R^2) \qquad (3)$$

where the function $U(\varepsilon_A)$ is the area strain energy per unit area, potential energy of loading = -Fh,

where F is the weight applied.

Equilibrium will occur at the minimum value of the total energy $\pi R^2 U(\varepsilon_A) - Fh$.

In particular instances, there may be special geometrical constraints to be included. Furthermore there may be other energy terms to be included in the sum such as those due to compression of the underlying substrate and to frictional resistance to relative movement between substrate and fabric. Whereas, in the simple situation, the minimum energy would occur with a maximum value of R, since this minimises the strain, the other terms may serve to limit R. However, such details go beyond the scope of this paper, which is only concerned to demonstrate the significance of resistance to area change as a fabric property. What is clear is that a knowledge of values of $U(\varepsilon_A)$ would be of greater value to the design engineer, than other information on fabric mechanical properties. In many instances, this knowledge would enable any problem to be solved to an acceptable accuracy, although one must recognise that in some circumstances, such as those illustrated in Figure 6, the required area change could only occur when the necessary changes in shape are not prevented by the constraints on the system.

The above discussion relates to displacement under load, which may be of direct relevance, particularly when it leads to the development of corrugations. However, the analysis could be carried further into situations where failure occurs when the area change becomes so large that the fabric bursts. There will also be energy absorption associated with the change; this may be a nuisance, increasing the power consumption of vehicles or even slowing them to a halt, or it may be an asset as in the absorber function listed by Giroud (2b).

Area change and conformability

Any thin sheet material will have low bending stiffness and be flexible. But, as discussed in a preliminary account of current research (2c), double curvature into rounded surfaces requires an associated area change. If this area change is resisted, then any attempt to bend simultaneously in more than one direction will force a material into sharp point discontinuities. This happens with paper, but not with woven or knitted fabrics. Nonwoven fabrics occupy an intermediate position.

Some measure of conformability is required in geotextiles, in order to give ease of handling, to allow the fabric to mould itself on rough surfaces or to fit round corners, and to avoid the damage which can occur at sharp points. In order to achieve this desirable behaviour, a mode of easy area change is required. It might seem that this is in direct conflict with the requirements for resistance to area change in order to give support, but a suitable compromise may result from differences in response in tension and compression, or at high and low strains: a high area modulus is needed under high tensile stress, but not necessarily in compression or under low tensile stress.

The interaction with permeability

Associated with area change, there will be a change in the space within a fabric, and thus in its permeability. Consequently there will be significant interactions between area change and the various hydraulic functions of geotextiles.

MEASUREMENT OF FABRIC PROPERTIES

The problem

Although the concept of area change is theoretically as simple as that of volume change in a solid, the practical problem is that it is difficult to apply a uniform biaxial tension as in figure 2(b), because any grip needed to impose extension in one direction impedes extension in the other direction, and it is impossible to apply compression as in figure 2(c) without the specimen buckling. There is thus no direct way of determining the response shown in figure 3(c).

It is necessary to use other test methods and then calculate the area response functions. However the eleven tensile test methods shown by Giroud (2b), and others, involve a variety of combined strains and need to be analysed carefully.

Linear, elastic behaviour

In a linear elastic material at small strains, the three independent in-plane directions lead, in the most anisotropic material, to six independent elastic constants, and the area modulus will be a function of these parameters. In practice, for orthogonal materials it may be easiest to measure tensile moduli and Poisson's ratios in three directions and then calculate the required quantities. Alternatively shear moduli can be found.

A full account of the theory will be published elsewhere, but some useful results are given here. For linear elastic behaviour, the energy of fabric deformation and the area strain are related by:

$$U = \tfrac{1}{2}\ \text{Vol. Am.}\ \varepsilon_A^{\,2} \qquad (5)$$

where Am is defined as the effective area modulus which is a function of loading ratio along two normal directions (w) and the elastic constants of the fabric.

The values of Am for the state of hydrostatic stress (n=1, corresponding to the minimum energy per unit area strain) and the case of uniform circumferential strain $\left(n = \frac{E_1(1+\mu_2)}{E_2(1+\mu_1)}\right)$ can be shown to be:

Hydrostatic stress
$$Am = \frac{E_1E_2}{E_1(1-\mu_1)+E_2(1-\mu_2)} \qquad (6)$$

Uniform strain
$$Am = \frac{E_1(1+\mu_2)+E_2(1+\mu_1)}{4(1-\mu_1\mu_2)} \qquad (7)$$

where E_s and μ_s have their usual engineering meaning.

With regard to the denominator of equation 7, it should be emphasized that the product of the Poisson's ratios can not exceed unity and the modulus never assumes a negative value.

Non-linear and inelastic behaviour

When the material response is non-linear, there can be no simple relations of the type given in the last section. With three principal strains, $\varepsilon_1, \varepsilon_2, \varepsilon_3$, one has:

$$\text{area strain} = \varepsilon = f_1(\varepsilon_1,\varepsilon_2,\varepsilon_3) \qquad (8)$$

Equation (8) can be derived explicitly from the strain geometry, but its form depends on the particular strain definitions adopted.

$$\text{specific energy of deformation} = U = f_2(\varepsilon_1,\varepsilon_2,\varepsilon_3) \qquad (9)$$

Equation (9) must be determined experimentally, or calculated theoretically from structural mechanics, since it represents the material properties.

Any chosen value of ε_A will define a set of values of $(\varepsilon_1,\varepsilon_2,\varepsilon_3)$ and it is necessary to find which combination gives the minimum value of U. This minimum would be the value of $U(\varepsilon_A)$, which could be used directly in equations such as (3). Alternatively, we could plot the stress-strain curve by using the relation:

$$\text{effective hydrostatic stress} = dU(\varepsilon_A)/d\varepsilon_A \qquad (10)$$

In practice instead of being forced to carry out the complete search to find the precise minimum, it may be better to study the relations between area change and deformation energy in various particular directions. In addition, the multi-valued character of the functions must be examined when hysteresis or time-dependence are material properties.

Experimental methods

Errors due to edge effects, to non-uniform deformation, and to fabric buckling are the bane of textile testing, and it is not possible to mention all the problems in this short paper. Only a broad outline of methods can be given.

Simple uniaxial extension, carried out on a standard laboratory tensile tester, can be adopted to study area change if the lateral contraction is measured. It is also preferable to measure the axial strain in the central region of uniform deformation. A satisfactory but time-consuming method, which we have adopted, is to draw a square on the fabrics and then photograph this at successive known stages of the tensile test. By cutting samples at different angles and stretching in the bias direction it is possible to determine the area changes associated with shear as well as those due to extension along the principal axes of the fabric.

One special experimental trick should be noted. If a single piece of fabric is extended on the bias it exerts a sideways pull on the grips. This can be avoided by mounting together two samples facing in opposite directions.

Further development along this approach to the problem would involve the direct study of fabric shear (with specimens cut in different directions), the construction of a suitable biaxial tester, and, most important, the search for a convenient automatic method of monitoring the strains.

An alternative approach to the problem is to go directly for some experimental arrangement akin to figure 4. A laboratory bursting tester would be one means. A penetration tester with a plunger pressed on to a clamped fabric specimen is another which we have used in other studies (4). In current work a ball penetration tester is being used.

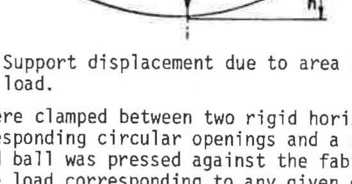

Figure 4. Support displacement due to area change under load.

Fabrics were clamped between two rigid horizontal plates with corresponding circular openings and a symmetrically positioned ball was pressed against the fabric. A curve giving the load corresponding to any given depth of deformation (δ) can be obtained from the CRE machine.

The curve can be used to determine the energy for any given value of δ which in turn is used to calculate the area strain from the geometry.

By selecting a suitable ball size, too large to penetrate the fabric but sufficiently small to deform the fabric into conical rather than a spherical shape, the fabric deforms under uniform circumferential strain corresponding to equation 7. The area modulus thus obtained can be compared with the results of the simple tensile tests along different directions.

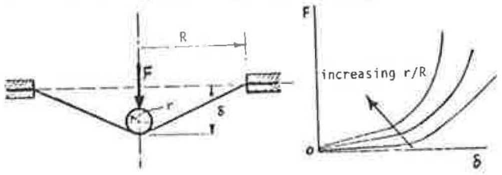

Fig.5(a) Schematic experimental set up.
(b) Typical load-deformation curves.

Experimental results

We are not in a position to give comprehensive data on the range of fabrics of interest in geotextiles. Indeed, since the research is being carried on in a broader context of fabric mechanics related to more traditional uses of textiles, the particular fabrics so far studied are not likely candidates for engineering use. Nevertheless some experimental results may serve to indicate general trends.

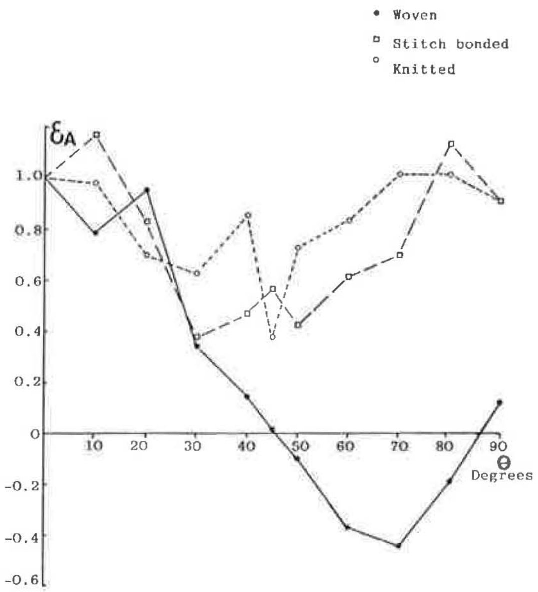

Fig.6 Area strain per unit tensile strain along different directions.

The experimental results given in figures 6 and 7 are obtained from uniaxial tests using 40 x 40 cm samples. The strains were measured by photographic means and the corresponding energies calculated from the CPE machine charts. The values given are relative to the corresponding values along direction θ=0 (warp direction, machine direction and wale direction)

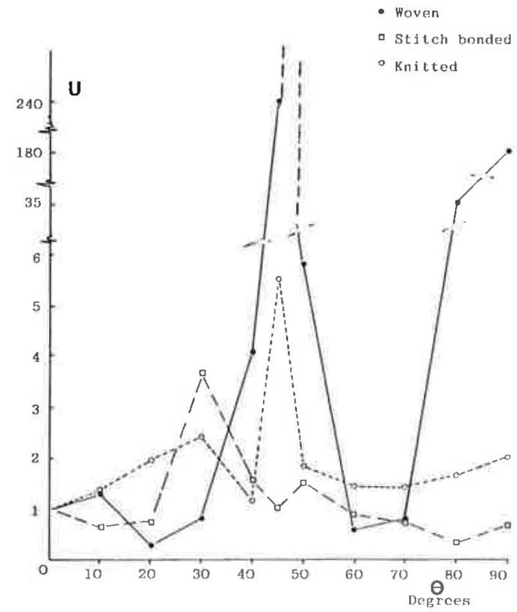

Fig.7. Energy of fabric deformation per unit area strain along different directions.

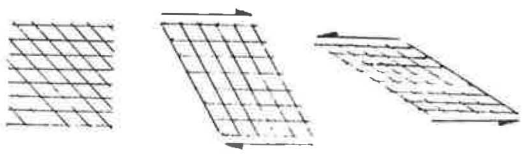

Fig. 8. An oblique lattice which can change area by shear.

CONCLUSION

The anelastic properties of fibre materials, the slippage between fibres in assemblies, and the anisotropic structure of fabrics makes the complete specification of their mechanical properties a formidable task. It is easy to take refuge in tests which are simple to perform, but often these have little direct relevance to behaviour in geotextile uses, although they may be of some value as broad indication of "strength" and are always useful as a means of monitoring the maintenance of quality. Despite the difficulties, a satisfactory treatment of the fabric mechanics is a prerequisite for real geotextile engineering.

The underlying form of the general relations, discussed in the first part of this paper, is a necessary starting point in the search for simplification. We have concentrated attention on resistance to area change which can be seen as an important mechanism in any support function of geotextiles, while conversely ease of area deformation is needed to ensure conformability. In the best materials non-linearity of response will give a good combination of properties.

The experimental results quoted illustrate the fact that in all three fabrics tested, the ease of area increase under tension reduces along directions other than the principal ones.

The woven fabric shows a reduction in area at angles greater than 45 degrees, which is a result of the high Poissons ratios along these directions.

The energy of fabric deformation per unit area change increases dramatically around 45 degrees for knitted and woven fabrics (in the latter, with regard to zero area change between positive and negative values, the energy approaches infinity), while the peak for stitch bonded fabric appears around 30 degrees corresponding to the minimum ε_A per unit tensile strain.

References

(1) Shanahan, W.J. Lloyd, D.W. and Hearle, J.W.S. Textile Res.J.,48, 495-505 (1978).

(2a) Lloyd, D.W. pages 11-342; (2b) Giroud, J.P. pages 513-534; (2c) Hearle, J.W.S. and Amirbayat, J. pages 570-574 in Mechanics of flexible fibre assemblies, edited by J.W.S. Hearle, J.J.Thwaites, and J. Amirbayat, NATO Advanced Study Institutes Series E, No.38, published by Sijthoff and Noordhof, 1980.

(3) Ford, H. Advanced mechanics of materials, 2nd edition, Ellis Horwood, 1977, page 125.

(4) Sultan, M.A.I. unpublished reports on research at UMIST, sponsored by U.S. Army E.R.O.