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Search for critical failure lines of models reinforced with rockbolts and ground anchors

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ABSTRACT: We are developing the design technique for an underground space which is reinforced with rockbolts and ground anchors.

First, we carried out some model tests. In these model tests, models reinforced with rockbolts and ground anchors were used. The ground excavation was simulated by reducing only the lateral pressure acting on a model under the condition of plane strain. The location of the failure lines of the models and the failure loads were confirmed through these tests.

Secondly, each model test was simulated by using the FEM(finite element method). The local stresses of the model under the failure load which was confirmed by the model test were estimated. For each model, the failure line having the minimum safety factor was searched for and located in the FEM stress field through the employment of the Dynamic Programming approach.

The failure lines estimated by the aforementioned technique were very close to those confirmed by the model tests. In addition, the safety factors of the estimated failure lines were nearly equal to 1.0. As a result, it was found that this technique was useful for the design of underground spaces which were reinforced with rockbolts and ground anchors.

1 INTRODUCTION

In Japan, especially the Metropolitan area (around Tokyo), a lot of structures have been built and it is difficult to build additional structures because of a lack of land which can easily purchased. However, in order to let the city function properly, the demand of additional public facilities is increasing.

In this situation, underground space is needed, where public facilities (railways, roads, substations for the electric power supply, incinerator plants, sewage disposal plants, etc.) are built.

In the south western part of the Metropolitan area, we can find mud stone from an area which begins at 50m below the surface of the ground. Its unconfined compression strength is from $2000kN/m^2$ to $7000kN/m^2$.

We are developing a method which employs rockbolts and ground anchors to construct underground space in this kind of mud stone (see Fig.1).

In order to put this construction method to practical use, we have to develop the design technique of the underground space

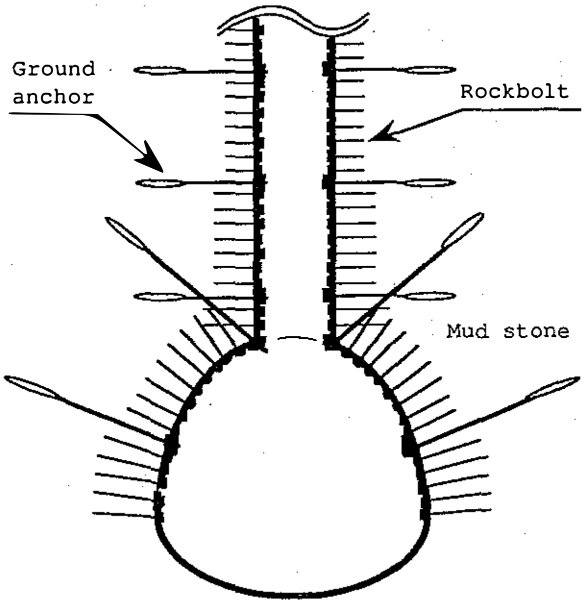
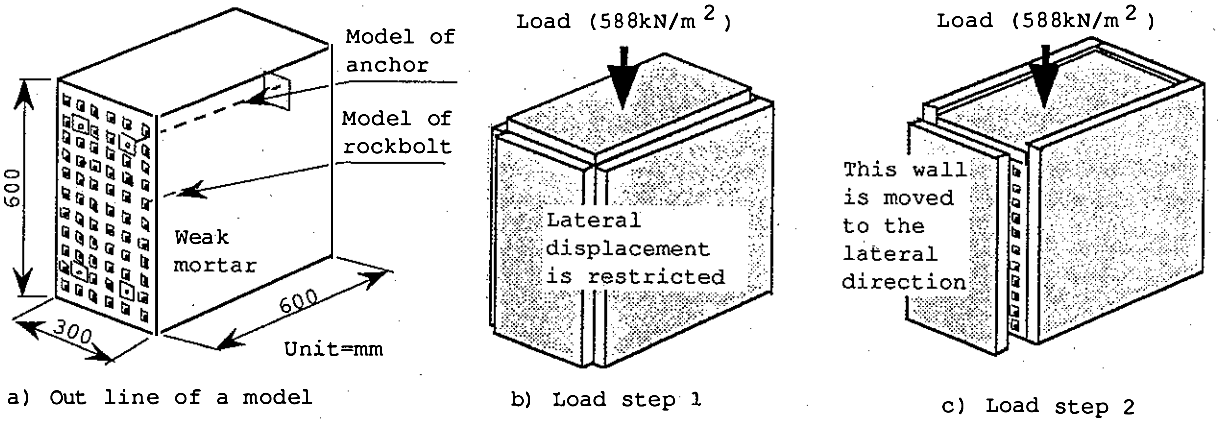


Figure 1 Proposed method to construct an underground space by employing rockbolts and ground anchors



a) Out line of a model b) Load step 1 c) Load step 2

Figure 2 Outline of the model test

considering the effect of rockbolts and ground anchors.

The shape of the underground space is not simple and the stress values around the reinforcements are very complicated. So, an FEM analysis has to be performed to estimate the local stress in the ground. However, the stress values obtained from the FEM do not directly show the stability condition of the underground space.

On the other hand, although limit equilibrium methods show the failure modes and the overall factor of safety of the underground space, it is impossible to evaluate the local stress accurately by these methods.

Therefore, we combine the advantages of both approaches. We utilize the stress value obtained from the FEM within the framework of the limit equilibrium method. For this purpose, the same procedure which was proposed by Yamagami (Yamagami 1988) is applied. In this procedure, the critical failure line in the FEM stress field is searched for and located by employing a Dynamic Programming approach.

In this paper, the model tests will be performed to confirm the location of the failure lines of the models and the magnitudes of the load at failure. Then, the comparison of the analyzed results with experimental results will be performed to examine the appropriateness of the procedure mentioned above.

2 OUTLINE OF THE MODEL TEST

Beforehand, we had performed loading tests and examined our analysis technique (Tsubouchi 1991). In this loading tests, only the vertical pressure acting on the model surface was increased.

At this time, both vertical and lateral pressure were increased to simulate the initial condition of the ground. After that, only the lateral pressure was reduced to simulate the ground excavation. Fig.2 shows the outline of this test. The model used in this test simulated a part of Fig.1.

Rockbolts or ground anchors were set into a model formed with weak mortar (unconfined compressive strength=117.6kN/m ^{2}). The anchors had rear plates to simulate the perfect fit between the anchors and the model. Table 1 shows the material properties of the model.

The load was applied on the top of the model under the condition that lateral displacement did not appear (see Fig. 2b). By means of this operation, the stress condition of the ground before excavation was simulated. After that, one wall was moved to the lateral direction in order to reduce the lateral pressure (see Fig.2c). By means of this operation, the stress condition of the ground after excavation was simulated. Through these operations, the condition of the plane-strain was preserved.

Four different combinations of rockbolts and anchors were tried including an unrein-

Table 1 The material properties of model

Materials	Items	Values	
Weak mortar	Young's modulus	12.35	(MN/m^2)
	Poisson's ratio	0.25	
	Unit weight	16.4	(kN/m^3)
	Cohesion	72.52	(kN/m^2)
	Internal Friction angle	37	(deg)
Rockbolt	Young's modulus	70300	(MN/m^2)
	Diameter	3	(mm)
Anchor	Young's modulus	70300	(MN/m^2)
	Diameter	9	(mm)

forced model. Table 2 shows the test cases.

The failure lines confirmed in each test are shown in Fig.3. In Fig.3, the following conclusions are established.

1. If we use only ground anchors, the failure lines appear at the surface between the anchors (see Fig.3b).
2. If we use only rockbolts, the failure lines appear in the inner part of the model(see Fig.3c).
3. If we use rockbolts and anchors together, the failure lines at the surface and inner part of the model disappear (see Fig.3 d).

3 PROCEDURES OF SEARCHING FOR FAILURE LINES

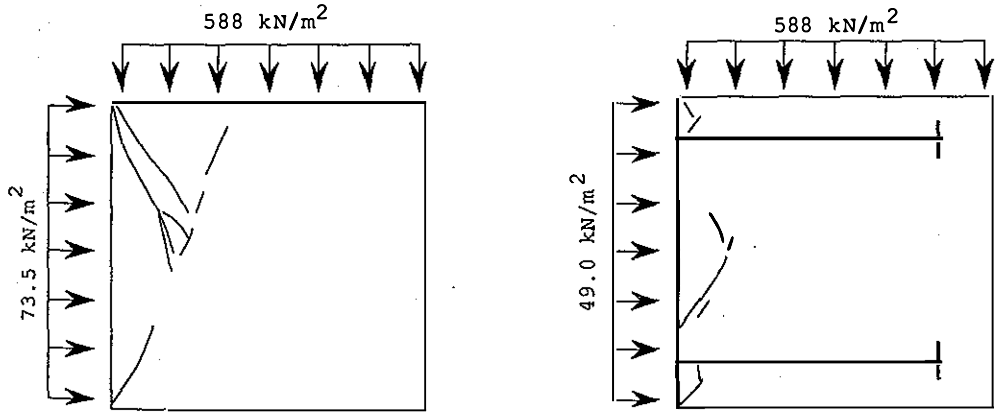
We simulated the model test mentioned above by the FEM and searched for the critical failure lines in the FEM stress fields. This process is expressed as follows.

First, the FEM model is established simulating the model test as shown in Fig.4 and the stress value of each element is calculated.

The failure line shown in Fig.4 is considered and its safety factor is defined by the following equation using Coulomb's failure criterion.

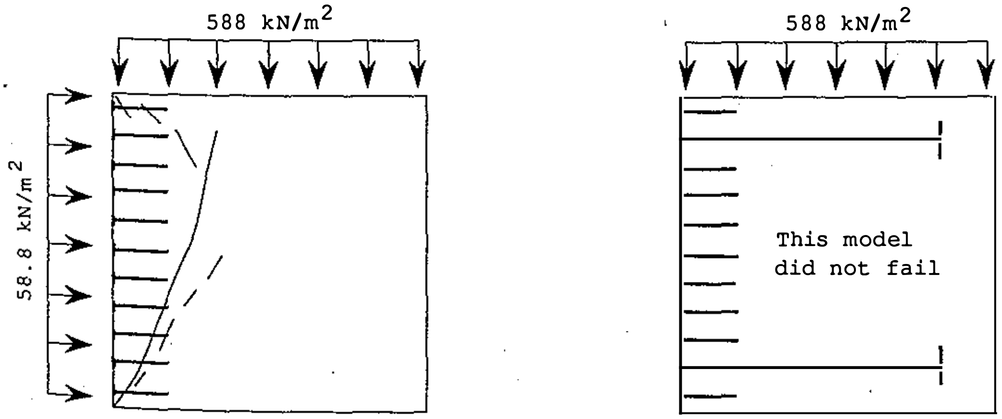
Table 2 The model test cases

Case	Rockbolt		Ground anchor	
	Length (mm)	Interval (mm)	Length (mm)	Interval (mm)
Case A (no reinforcements)	-	-	-	-
Case B (only anchors)	-	-	500	300 x 150
Case C (only rockbolts)	100	50 x 50	-	-
Case D (anchors & rockbolts)	100	50 x 50	500	300 x 150



a) Case A (no reinforcements)

b) Case B (only ground anchors)



c) Case C (only rockbolts)

d) Case D (anchors and rockbolts)

Figure 3 The failure lines confirmed by model test

$$Fs = \frac{\int_s (C + \sigma \cdot \tan\phi) \cdot ds}{\int_s \tau \cdot ds} \qquad (1)$$

Where

Fs : safety factor of the failure line

C : cohesion of the model

ϕ : internal friction angle of the model

σ : normal stress on the surface of the failure line

τ : shear stress on the surface of the failure line

The integrations are executed along the failure line.

There are numerous failure lines to be considered. The line which has the minimum safety factor is determined by using the optimization technique. Here, Dynamic Programming (Ogata 1973) is used. The Dynamic Programming approach resolves multistage optimization problems.

In order to apply Dynamic Programming to this problem, the appropriate number of stages in a given model has been established as shown schematically in Fig.5. At each stage the appropriate number of states has been provided, which are indicated by the points in Fig.5.

Now, we consider a failure line made by connecting points at two arbitrary successive stages as shown in Fig.5.

For this failure line, Eq.(1) is rewritten as

$$Fs = \frac{\Sigma R_i}{\Sigma T_i} \qquad (i=2,M) \qquad (2)$$

Where

$$R_i = \int_{s(j,k)} (C + \sigma \cdot \tan\phi) \cdot ds \qquad (3)$$

$$T_i = \int_{s(j,k)} \tau \cdot ds \qquad (4)$$

Here, $s(j,k)$ denotes the line connecting point j at the stage $i-1$ and point k at the stage i (see Fig.6). M is the total number of stages.

In the execution of Eq.(3) and (4), the stress value σ and τ obtained from FEM analysis are used. Here, it is assumed that stress is constant in each FEM element.

Further, we define the new auxiliary function G as

$$G = \Sigma (R_i - Fs \cdot T_i) \qquad (i=2,M) \qquad (5)$$

It is known that minimizing the function Fs in Eq.(2) is equivalent to minimizing the new function G.

According to the "principle of optimality", which is the central concept in Dynamic Programming, the minimum value of G between the initial stage and point k, the function $H_i(k)$, is given by the sum of the

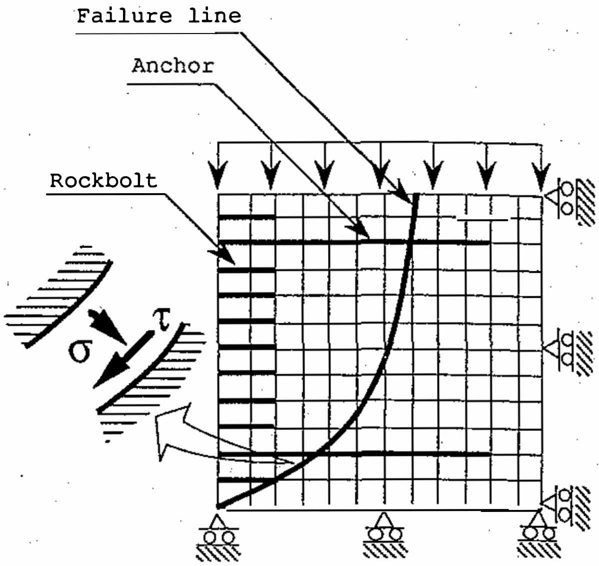


Figure 4 Definition of the safety factor

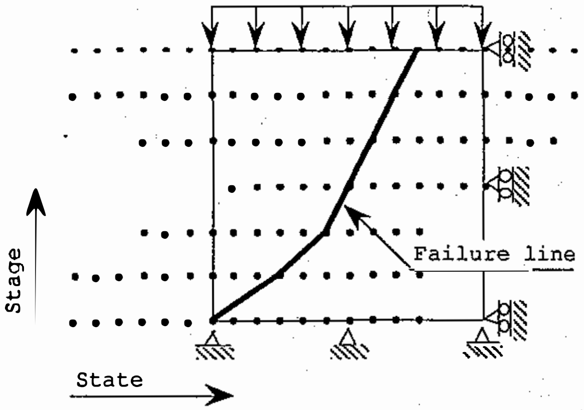


Fig. 5 Representation of stages and states

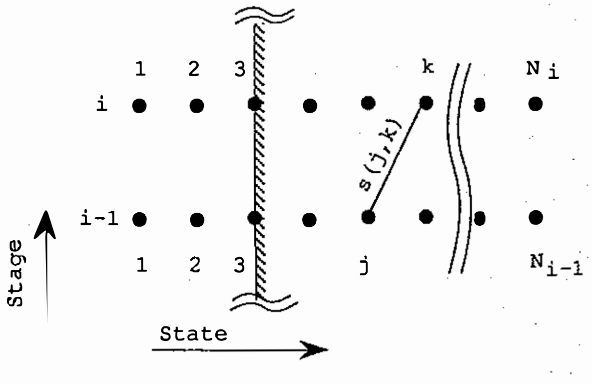


Fig. 6 Application of Dynamic Programming

minimum value of G between the initial stage and any state j at the previous stage i-1 and the change in G on passing between the two states j and k. This is expressed as

$$H_i(k) = \min_{1 \leq j \leq N_{i-1}} [H_{i-1}(j)+D_i(j,k)] \qquad (6)$$

$$(i=2,M) \quad (k=1,N_i)$$

Where, N_i is the number of states at stage i. $D_i(j,k)$ is the change in G on passing from the point j to the point k and is expressed as

$$D_i(j,k) = R_i - Fs \cdot T_i \qquad (7)$$

After the calculation of Eq.(6) reaches the final stage, the minimum value of G is obtained by the following equation.

$$G_{min} = \min_{1 \leq k \leq N_M} H_M(k) \qquad (8)$$

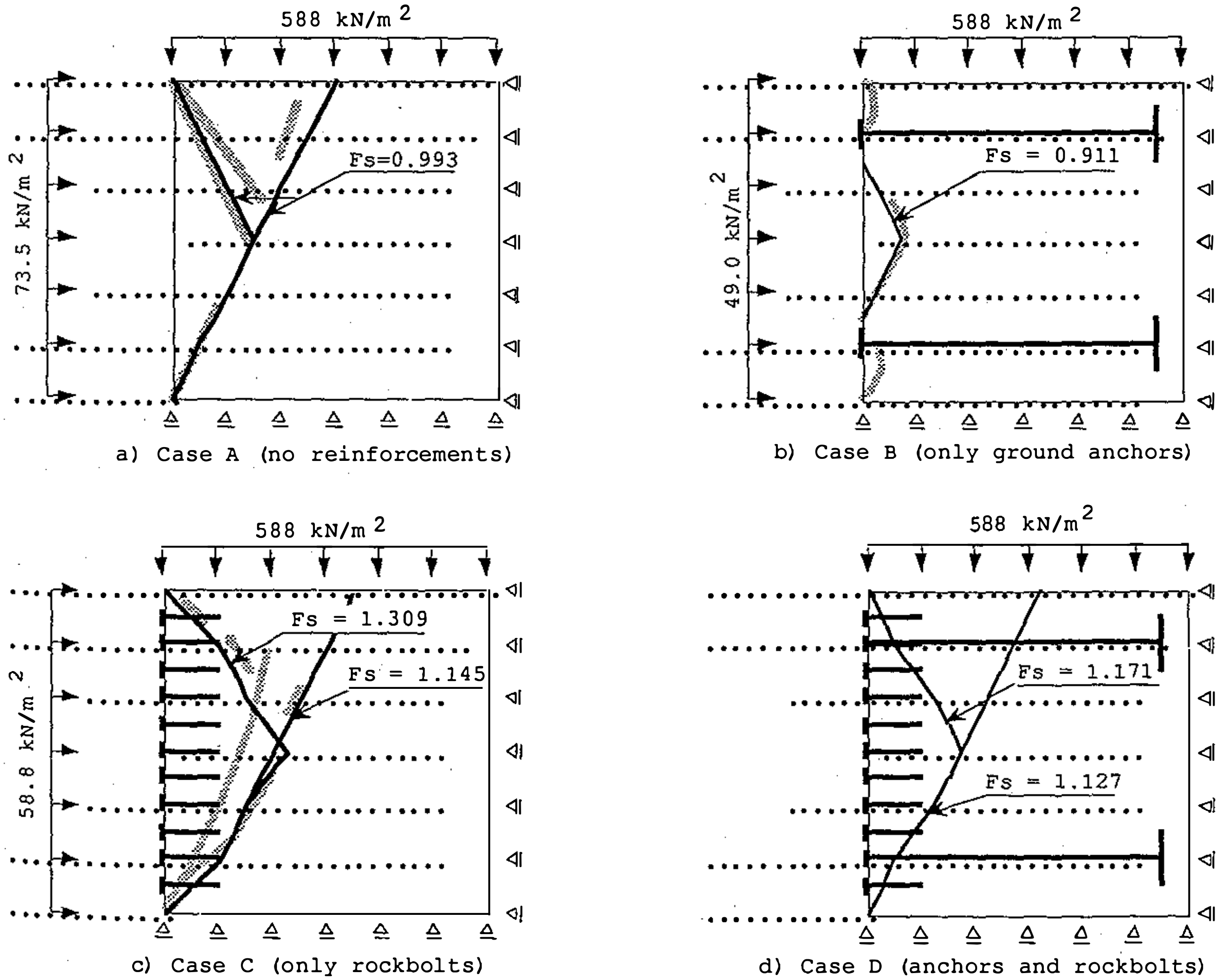
The critical failure line is located by tracing back the path which gives G_{min}.

4 COMPARISON OF THE ANALYZED RESULTS WITH THE EXPERIMENTAL RESULTS

We simulated all four model tests by the analysis mentioned in chapter 3. In this analysis, the failure load confirmed in each model test was applied. The material properties adopted for this analysis was the same as shown in Table 1. Rockbolts and anchors were treated as beam elements and it is assumed that there was perfect cohesion between the reinforcements and the models.

Fig.7 shows the comparison of the analyzed failure lines with the experimental ones.

In Fig.7a,b, it can be found that the failure lines located in the analysis are very close to those confirmed in the model tests



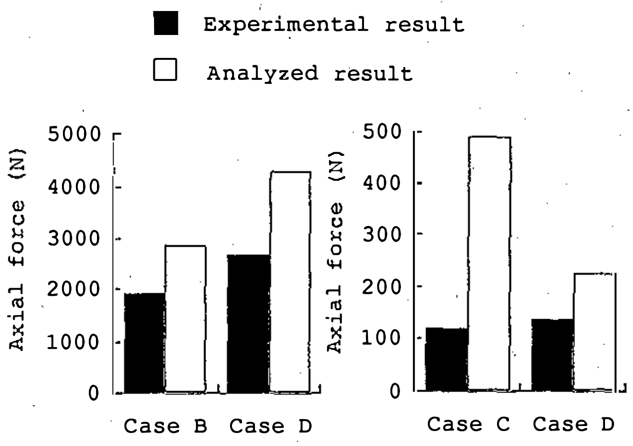
a) Case A (no reinforcements)

b) Case B (only ground anchors)

c) Case C (only rockbolts)

d) Case D (anchors and rockbolts)

Fig.7 Comparison of the analyzed failure lines with the experimental ones



a) Ground anchor force b) Rockbolt force

Figure 8 Comparison of the analyzed results with the experimental results on the axial forces of reinforcement

and their safety factors are less than 1.0. So, it can be estimated that these models will fail.

In Fig.7d, the safety factor is bigger than 1.0. So, it can not be estimated that this model will fail.

Concerning these cases(Case A,B,D), analytical results and experimental results concur with each other.

In Fig.7c, the failure lines located in the analysis are very close to those confirmed in the model tests. However the analytical safety factor is bigger than 1.0. So, the analytical result and the experimental result do not concur with each other.

In the analysis, it is assumed that the cohesion between the rockbolts and the model is perfect. However, in the model tests, the cohesion is not perfect.

Fig.8 shows the comparison of analyzed results with experimental results in the axial forces of the reinforcements.

It is found that the analyzed axial forces of the reinforcements are greater than the experimental ones. This trend is especially noticeable in Case C.

It can be considered that the effect of the rockbolts is overestimated in the analysis of Case C. This is the reason that the analyzed safety factors of the failure lines are greater than 1.0 in Case C.

To apply this analysis as the design technique for utilizing underground space, the sliding that occurs between rockbolts and ground should be considered. Otherwise, the effect of the rockbolts will be overestimated.

5 CONCLUSIONS

We are developing a method which employs the rockbolts and ground anchors to construct an underground space. In our work, we need the appropriate design technique for this construction method.

The procedure of searching for the failure line having the minimum safety factor in the FEM stress field was adopted to express the failure line obtained from the model test.

As a result, the following was found: the aforementioned technique could simulate the experimental failure lines, and the safety factor of the analyzed failure lines was nearly equal to 1.0. Therefore, it can be concluded that this technique has the probability to be used in the design stage of an actual underground space.

When this technique is used in the design stage of an actual underground space, it has the following advantages.

1. It incorporates the interaction between the reinforcements and the ground.
2. It evaluates the local stress in the ground with relative accuracy.
3. It determines the overall factor of safety of the underground space.
4. The calculation in this technique is so easy that it can be performed by a micro computer. (Actually, all calculations appearing in this paper were performed using a micro computer and it took only 5 minutes to calculate each case.)

We are going to consider this technique as one of the design tools of an underground space which is reinforced with rockbolts and ground anchors.

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