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Settlement reduction due to extensible reinforcement strip

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ABSTRACT:The elastic continuum approach is used to model the soil-extensible strip interaction. The shear stresses mobilised at the interface due to the lateral movement of the soil and the resulting reduction in settlements of the points on the surface are computed. A parametric study brings out the effects of the length and depth of the strip, and the aspect ratio of the loaded area, on the reduction in surface settlements. Correction factors to estimate the reduction in settlements in terms of those for rigid strip are proposed.

1 INTRODUCTION

Reinforced foundation beds show remarkable improvement in the bearing capacity and in the reduction of settlements, if reinforced with strips, grids, or cells. Few analytical solutions are available to predict the increase in the bearing capacity of reinforced beds (Binquet and Lee 1975, Giroud and Noiray 1981, Ingold and Millar 1982, Sridharan et al 1988, etc.). Finite element studies (Brown and Poulos 1981, Andrawes et al 1982, Floss and Gold 1990, etc.) help in understanding the behaviour of reinforced soils. A number of laboratory scale model tests (Love et al 1987, Dembicki and Alenowicz 1988, etc.) are also available. The literature mainly deals with the bearing capacity improvement of soils due to the reinforcement. The present study applies the elastic continuum theory to predict the shear stresses mobilised at the interface of an extensible strip and the soil, and subsequently the reductions in the surface settlements.

2 PROBLEM DEFINITION AND MECHANISM

An extensible strip of size, $2L_r x 2B_r$, and at depth, U_0, placed centrally beneath a loaded surface area of size, $2B_f x 2L_f$, transmitting a load of intensity, q^f, (Fig.1), is considered. The width, $2B_r$, of the strip is relatively small while its thickness, t_r, is negligible. The surface load causes the soil to displace vertically and laterally. In this analysis, the deformation of the strip in the longitudinal direction alone is considered. It is assumed that the displacement of the strip in the vertical direction does not affect the shear stresses mobilised at the interface. The soil is assumed to be homogeneous, isotropic, and linearly elastic. The lateral movement of the soil at the interface is resisted by the shear stresses mobilised. These stresses in turn push the soil above upward thus resulting in the settlement reduction of the points on the surface.

3 FORMULATION

The horizontal displacement of point i (Fig.1) along the length of the strip is calculated by integrating Boussinesq's equation numerically over the loaded area. For this purpose, the loaded area is subdivided into $n_b x n_l$ subareas. Due to symmetry, only N elements each of length dl, along half the length of the strip are considered. The details of the integration procedure are given in Madhav and Pitchumani (1991). The vector of horizontal displacements of the N nodes along half-length of the strip due to surface load, q, is

$$\{\rho_x^f\} = \frac{B_f}{E_s} \{ I^f\}q \qquad (1)$$

where $\{ \rho^f_x\}$ and $\{I^f\}$ are vectors of size N. The vector $\{I^f\}$ is a function of the aspect ratio, L_f/B_f, of the loaded area,

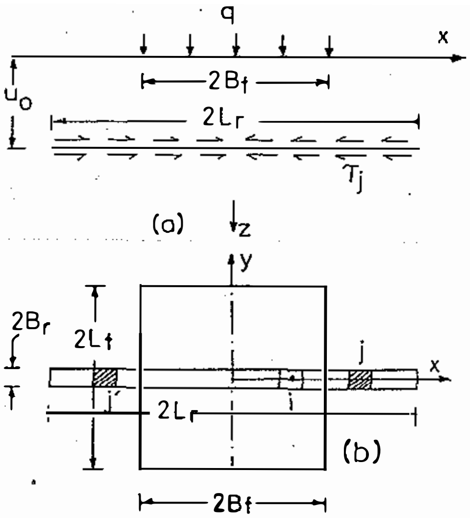

Fig.1 Definition sketch

the depth of the strip, U_0/B_f, and the Poisson's ratio, ν_s, of the soil.

The horizontal displacements of the N nodes due to the shear stresses(τ) mobilised at the interface are evaluated by integrating Mindlin's solution for the displacements due to a horizontal force within the continuum. While performing the integration, it is assumed that the shear stress (τ), over each element is constant. The vector of horizontal displacements of the N nodes due to shear stresses is

$$\{\rho_x{}^r\} = \frac{B_f}{E_s}\,[\,I^r\,]\{\tau\} \tag{2}$$

where the vectors $\{\rho^r{}_x\}$ and $\{\tau\}$ are of size N. $[I^r]$ is a coefficient matrix for the influence of the shear stresses on the horizontal displacements. It is a square matrix of size N and depends on the length of the strip, L_r/B_f, depth, U_0/B_f, and the Poisson's ratio, ν_s, of the soil. For an extensible strip, the net lateral soil displacements are equal to the elongations of the nodes of the strip. Thus, the compatibility equation is

$$\{\rho_x{}^f\} - \{\rho_x{}^r\} = \{\rho_e\} \tag{3}$$

where $\{\rho_e\}$ is a vector of elongations at the nodes of the strip and is of size N. The elongations are due to the tension in the strip resulting from the mobilised shear stresses. The tension T_k to the left of element k, is

$$T_k = \sum_{i=k}^{N} T_i = \sum_{i=k}^{N} \tau_i . dl . 2B_r \tag{4}$$

The elemental elongation, $\Delta\rho_{ek}$, of the k^{th} element is

$$\Delta\rho_{ek} = \left(T_k + \frac{\Delta T_k}{2}\right)\frac{dl}{A_r E_r} \tag{5}$$

where $A_r = t_r x 2B_r$, is the cross-sectional area of the strip, and E_r the modulus of elasticity of the strip. Substituting Eq.4 in Eq.5, one gets

$$\Delta\rho_{ek} = \left(\frac{\tau_k}{2} + \sum_{i=k+1}^{N}\tau_i\right)\frac{dl^2}{t_r E_r} \tag{6}$$

The total elongation of node k is equal to the sum of all elongations of nodes 1 to k, and is

$$\rho_{ek} = \sum_{j=1}^{k}\Delta\rho_j \tag{7}$$

Combining Eqs.6 and 7, the vector $\{\rho_e\}$ is

$$\{\rho_e\} = \frac{dl^2}{t_r E_r}\,[I_e]\,\{\tau\} \tag{8}$$

where $[I_e]$ is a square elongation coefficient matrix of size N. Nondimensionalising all the length parameters with the parameter B_f, Eq.8 is written as

$$\{\rho_e\} = \frac{B_f (dl/B_f)^2}{t_r/B_f \cdot E_r}\,[I_e]\{\tau\} \tag{9}$$

Substituting for $\{\rho_e\}$ in Eq.3 one obtains

$$[\,[I_r] + \frac{m}{K}[I_e]\,]\{\tau/q\} = \{I^f\} \tag{10}$$

where $m=(dl/B_f)2$ and $K = E_r t_r / E_s B_f$. K is the relative elongation ratio which takes into account the axial stiffness of the strip and the modulus of deformation of the soil. Eq.10 gives N equations for the normalised shear stresses, τ/q, mobilised along the strip. These equations are solved using the Gauss elimination technique. The reduction in surface settlement due to these mobilised shear stresses is calculated using Mindlin's solution for the vertical displacements due to a horizontal force within an elastic continuum. The vertical displacement of node k on the surface due to shear stress, τ_j, is obtained by integrating Mindlin's equation numerically over the area of element j, as

$$\rho_{zkj}^{r1} = \frac{B_f}{G_s} I_{skj}^{r1} . \tau_j \quad (11)$$

where I_{skj}^{r1} is a dimensionless influence coefficient that depends on the parameters L_r/B_f, U_0/B_f, ν_s and the location of the node k and the element j. For every element j, there exists its image j' the shear stress on which is the same but acts in a direction opposite to that acting on element j. The influence of the stress acting on element j' on the vertical displacement of node k is

$$\rho_{zkj}^{r2} = \frac{B_f}{G_s} I_{skj}^{r2} . \tau_j \quad (12)$$

where I_{skj}^{r2} is the influence coefficient for the influence of the stress on element j' on the vertical displacement of node k. Combining Eqs.11 and 12, and obtaining the influence of the shear stresses on all the elements, the vertical displacement of node k is

$$\rho_{zk} = \sum_{j=1}^{N} \frac{B_f}{G_s} [I_{skj}^{r1} + I_{skj}^{r2}] \tau_j \quad (13)$$

The vector of vertical displacements, $\{\rho_z\}$, is

$$\{\rho_z\} = \frac{B_f}{G_s} [I_s^r]\{\tau\} \quad (14)$$

The vertical displacements are evaluated at N_f points along the width of the loaded area. The vectors $\{\rho_z\}$ and $\{\tau\}$ are of sizes N_f and N respectively while the matrix $[I_s^r]$ is of size N_fxN. Eq.14 is rewritten as

$$\{\rho_z\} = \frac{B_f}{G_s} \{I_s\}.q \quad (15)$$

The elements of the vector $\{I_s\}$ are termed as Settlement Reduction Coefficients (SRC) where

$$\{I_s\} = [I_s^r]\{\tau\} \quad (16)$$

4 RESULTS

The results of the parametric study are presented herein. The half-width of the strip is maintained at $0.05B_f$. For the purpose of numerical integration, the surface loaded area is divided into subelements of size $0.025B_f$x$0.025B_f$. Because of symmetry of loading Eq.1 is obtained by integrating the Boussinesq equation over only half the loaded area. The total effect is obtained by multiplying the influence coefficient thus obtained by two. The width of the elements, dl, along the strip is kept at $0.1B_f$ since elements of smaller size did not improve the results. In order to obtain the influence coefficients due to shear stresses on the horizontal displacements, element j is subdivided into 20x20 subareas for |i-j|<=2. For |i-j|>2 and for calculating I_{zkj}, the elements are divided into 4x4 subareas.The stresses evaluated are the total stresses mobilised on the top and bottom faces of the strip unlike in the case with Madhav and Pitchumani (1991) where only half the stresses (i.e. stresses mobilised on one face only) were presented. Further all the results presented are for Poisson's ratio of 0.3.

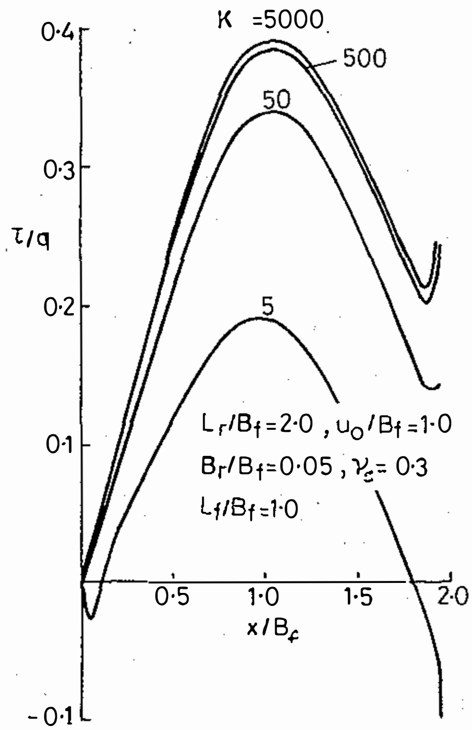

Fig.2 Effect of K on shear stresses

Fig.2 presents the variation of the normalised shear stress, τ/q, with distance, x/B_f, along the strip for different K values, and for a strip of length, L_r/B_f=2, placed below a square area (L_f/B_f=1) at a depth U_0/B_f=1. For all the values of the elongation ratio K , the shear stresses are a maximum at a distance x/B_f=1, i.e. below the edge of the loaded area. With increase in K values, the stresses increase and are positive through out the length of the strip. A positive shear

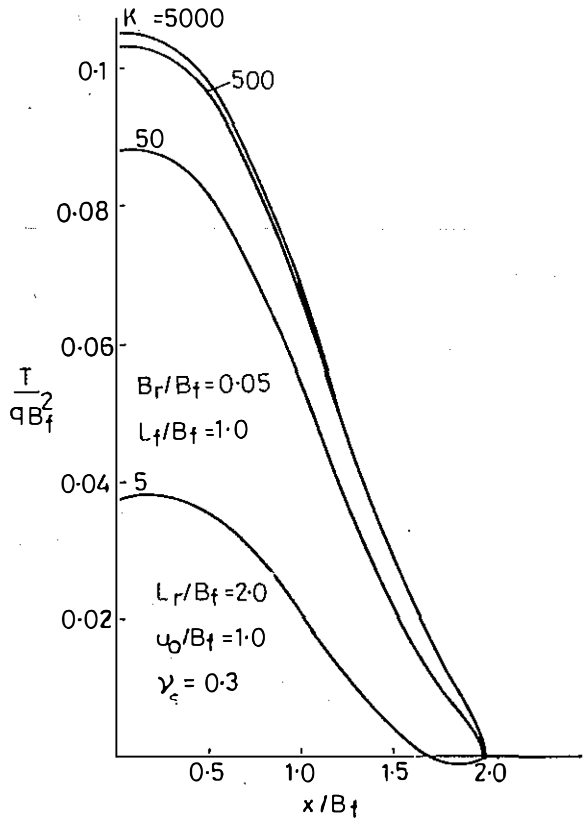

Fig.3 Effect of K on tension in the strip

stress is directed inwards and opposes the outward lateral movement of the soil. A strip with K =5000 behaves identically to a rigid one for both of which the maximum stress, τ/q, is 0.8.

The variation of the tensile force, T/qB_f^2, induced in the strip of length, L_r/B_f=2, placed below a square loaded area and at a depth, U_0/B_f=1, for various K values is shown in Fig.3. A rigid strip with K =5000 has a maximum tension of 0.105 qB_f^2 at the centre. For a highly extensible strip with K =5, the tension at the centre is only $0.038qB_f^2$, and near its edge a small compression is noted due to the mobilisation of negative stresses there at (Fig.2). The effect of K on the normalised elongation, E_s/qB_f, is depicted in Fig.4, for a strip of length, L_r/B_f=2, below a square loaded area, and at a depth U_0=B_f. A strip with K =5 shows a normalised elongation of the order of 0.067 at a distance, x/B_f=1.5, along the strip length. Beyond this distance, it decreases marginally as the strip is subjected to a small amount of compression in this zone. Elongation of the strip decreases with increasing values of K , as anticipated.

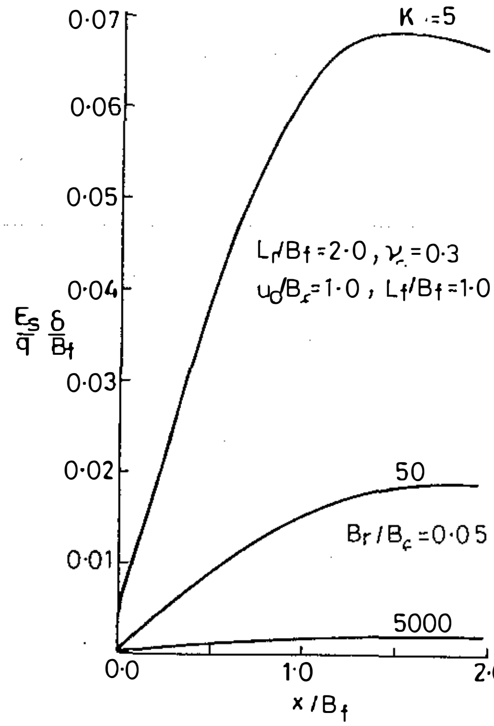

Fig.4 Effect of K on elongation of strip

The variation of SRC with distance, x_f/B_f, along the surface for various K values for a strip with length, L_r/B_f=2, placed below a square area at a depth B_f, is depicted in Fig.5. The SRC is maximum at the centre of the loaded area and decreases with distance from there.The SRC values at the centre and the edge of the loaded area for a highly extensible strip (K =5) are 0.0014 and 0.0001 respectively while those for a rigid strip (K =5000) are 0.0034 and 0.001 respectively. The increase in SRC values with K is a consequence of the increase in the shear stresses with K . The effect of the elongation ratio, K , on the settlement reduction coefficient at the centre of the loaded area I_{sc} (SRC at the centre), for various depth ratios, U_0/B_f, and for a strip with length, L_r=2B_f, placed below a square loaded area can be seen in Fig.6. I_{sc} is a maximum in the depth range of 0.75 to 1.0 B_f depending on the values of K .For a low value of K =5, the maximum I_{sc} value is 0.0015 for a strip at a depth U_0=0.75B_f which is around 0.45 times that for a rigid strip. As K increases, the I_{sc} values for strips at all depths tend to those for a rigid strip. At shallow depths U_0<0.25B_f, the I_{sc} values are practically zero indicating the ineffectiveness of the reinforcement considering only the effect of shear stresses.

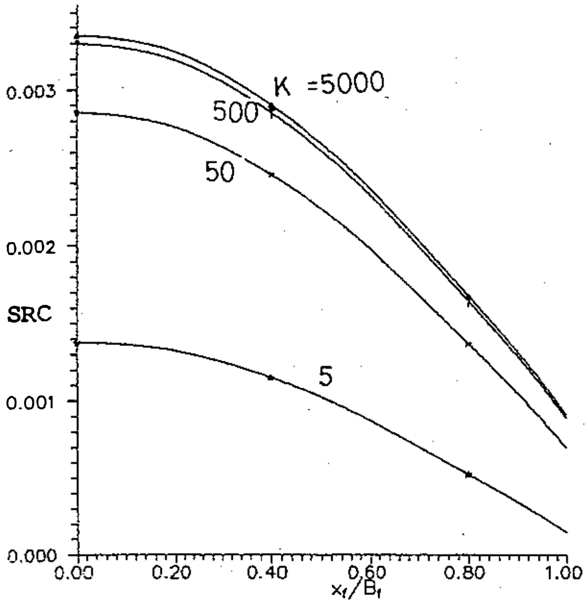

Fig.5 Effect of K on SRC
($L_r/B_r=2$, $L_f/B_f=1$, $U_0/B_f=1$)

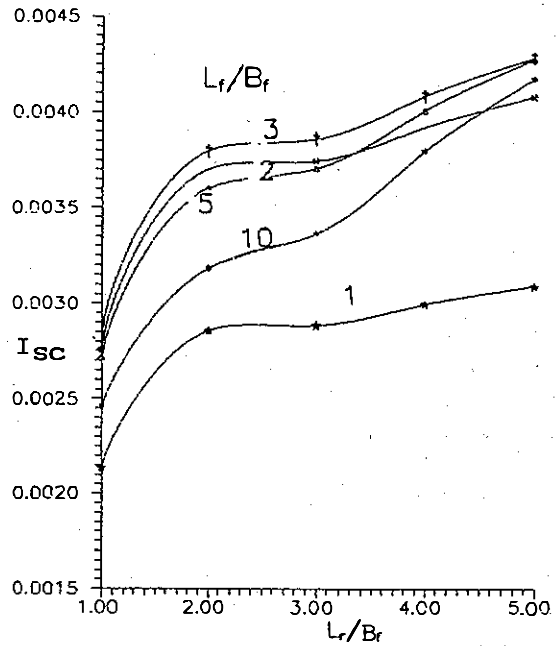

Fig.7 Effect of L_f/B_f on I_{sc}
($L_r/B_r=2.0$, $U_0/B_f=1$, K =50)

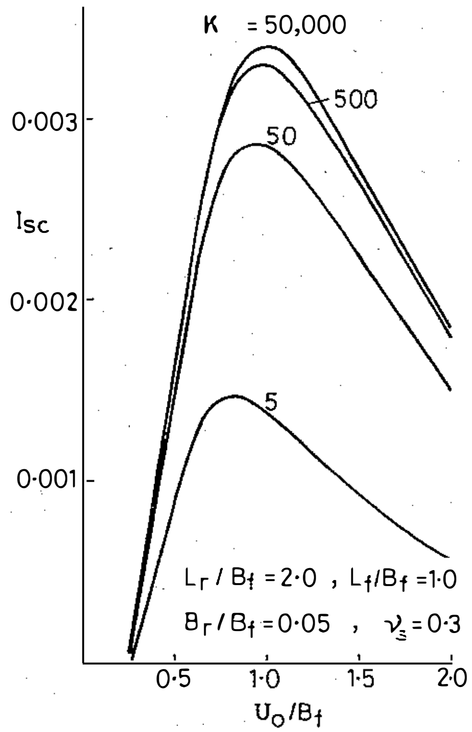

Fig.6 Effect of K on I_{sc}

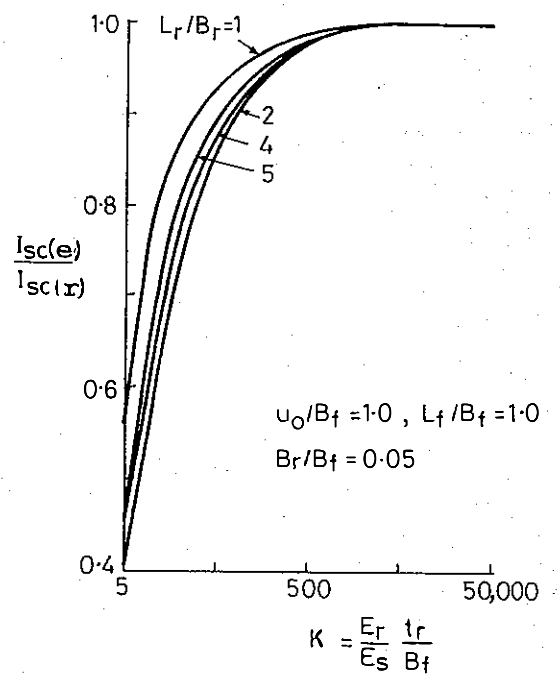

Fig.8 Correction factors for extensible strip

Fig.7 shows the plot of I_{sc} versus the length of the strip, L_r/B_f, for different aspect ratios, L_f/B_f, of the loaded area and for a depth ratio, $U_0/B_f=1$, and for K =50. For all lengths, L_r/B_f, I_{sc} values are the least for square areas. I_{sc} values are relatively high for strips below rectangles with $L_f/B_f>=3$. It is observed that I_{sc} values increase somewhat for lengths > $3B_f$ in case of long rectangles. This reiterates the fact that longer strips are effective below long rectangles. The effect of the length of the strip, L_r/B_f, on the ratio of I_{sc} for an extensible strip to that of a rigid one, $I_{sc}(e)/I_{sc}(r)$, for various K values and for a strip placed at depth B_f, is depicted in Fig.8. The ratio, $I_{sc}(e)/I_{sc}(r)$, increases with K and tends to a value of 1 for K =5000 indicating that the strip behaves as a rigid one. At low K values (K =5), the ratio is 0.56 and 0.4 respectively for strips of length B_f and $2B_f$. Based on this plot, I_{sc} values for extensible strips can be calculated from those for rigid ones. For strips with 100< K <500 and for K >500 correction factors of 0.9 and 1.0 respectively may be used for any length of the strip.

5 CONCLUSIONS

An elastic continuum approach is proposed to study the interaction between an extensible reinforcing strip placed below a uniformly loaded rectangular area, and the soil. The compatibility of lateral displacements of the points along the strip-soil interface is satisfied and the shear stresses mobilised computed. The reductions in settlements of the points on the surface due to these stresses are evaluated. The results obtained from the parametric study indicate that for maximum reduction in surface settlements due to shear stresses, the strips should be placed in the depth range of $0.75B_f$ to B_f where B_f is the half-width of the loaded area for strips of length equal to $2B_f$. Longer strips are preferable below long rectangles. For extensible strips, correction factors are given to arrive at the settlement reduction of the centre of the loaded area from the corresponding values for a rigid one.

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