POPESCU E.M.
Institut de Génie Civil, Roumanie

Some considerations on the design methods of reinforced earth retaining walls

Remarques sur les méthodes de dimensionnement de murs en terre armée

Au cours des dernières années, la terre armée a été largement utilisée pour la réalisation des murs de soutènement.
Le principe de la terre armée est bien simple, mais la prévision de son comportement par calcul s'avère plutôt compliquée.
On dispose maintenant d'un grand nombre de procédés, autant manuels qu'automatisés - par la méthode des éléments finis - pour calculer la stabilité intérieure ou extérieure d'un mur en terre armée. L'article passe brièvement en revue ces procédés, dont la validité est vérifiée par voie théorique et par confrontation avec les résultats des expériences sur modèles.

INTRODUCTION

The term of "reinforced earth structure" was introduced by Casagrande to represent the natural stratification of alternate soft and stiff material horizontal layers that commonly occurs in soil deposits (21).

More recently, an artificial reinforced earth was created by Henri Vidal (20). Reinforced earth is a soil mass composed of fill strengthened in critical directions by metal or plastic reinforcing strips. The material has been widely adopted in the civil engineering during the past few years.

The concept of strengthening soil with added rods or fibers is not new. Some animals and birds use straw and branches mixed with soil to build their habitations. The beneficial effect of plant roots in stabilizing soil has been recognized for a long time, and is recetly beginning to receive careful analytical study (8). Since Roman times, builders have been aware of the beneficiating effects of the inclusion of reinforcing elements in earthwork. A paper by Lee et al. (9) described some of the early concepts of reinforcing a soil as a construction technique.

In 1963 Vidal first published his results but it was not until 1965 that Vidal was able to design and construct a small reinforced earth wall at Prageres in the French Pyrenees. In 1968, the first large scale reinforced earth wall was built at Nice Menton in Southern France. This same year the full scale instrumented wall was constructed at Incarville, France (14). The successfull completion of this test has inspired the erection of several hundred reinforced earth walls in Europe (3, 13, 15), United States (5, 9, 10) and Japan (19) since then. In Romania the first reinforced earth walls were completed in 1973 in Iasi and Timisoara areas. A scale model investigation is being conducted by the Hydrotechnical Research Institute (4).

Because of its early use in road projects, reinforced earth is generally associated with retaining walls built to carry road beds over slides or valleys. But other applications include quay walls, dam or cofferdam works, bridge abutments and foundation rafts (15).

A cost comparison between reinforced earth, concrete and metal crib, and reinforced concrete walls indicated that reinforced earth walls offer a distinct economy for all wall heights (5). When foundation soil conditions suggest that considerable settlement may occur during the placing of backfill, reinforced earth construction offers an additional advantage over conventional walls. Since the reinforced earth wall is built up as the fill is placed, whereas the conventional retaining wall is cast first and then backfilled, the reinforced earth wall can take differential settlement better than conventional retaining walls.

REINFORCED EARTH RETAINING WALL ANALYSIS METHODS.

In considering reinforced earth for the purposes of design certain similarities with reinforced concrete can be identified. Like reinforced concrete, the beneficial effects depend on a combination of the tensile strength of the reinforcing and the shear bond with the surrounding soil. Reinforced earth is complicated by the fact that both the shear strength of the soil and the bond strength with the reinforcing are frictional in nature and are thus directly dependent on the normal effective stress distribution, which, in turn depends on the size, geometry, and type of loading of the structure as well as the types of materials, and other factors (9).

The major components of the reinforced earth walls are soil backfill, reinforcing flat strips or ties, and face covering skin elements (fig. 1a).

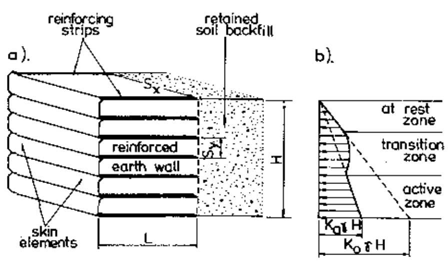


Fig. 1 - Major elements of reinforced earth wall and probable earth pressure distribution.

Because the reinforcing action requires good frictional bond between the ties and the soil, only free draining granular soils are considered, and the strips are oriented with the wide side horizontal.

In recent years research was conducted to adapt computer finite element techniques to more accurately portray the true mechanism of reinforced earth (7). There are two general approaches to the finite element analysis of reinforced soil systems involving respectively composite and discrete or dense tie-back representations of the constituents.

In a composite representation (6, 11) it is assumed that if the reinforcing pattern is repeated a sufficiently large number of times, the material can be considered homogeneous at the structural level (or as an inhomogeneous material in which the changing properties are due to changes in the reinforcing spacing and/or properties). The reinforced material when viewed, at the composite level, exhibits orthotropic behaviour. The composite properties assigned to the continuum elements reflect the properties of the matrix material and the reinforcing members and their composite interaction. Once the appropriate composite properties are determined, standard finite element procedures are used to analyse complicated structures of the reinforced material.

The advantage of a composite representation is the economy of analysis achieved by not having to discretely represent each and every reinforcing member. The disadvantage is that the analysis does not directly yield detailed information about the stress and strain states at the interfaces of the soil and the reinforcing members nor about localized deformations near the edges of the reinforced mass.

In a discrete representation (1, 16) the reinforced system is treated as a heterogeneous body, and the soil and each and every reinforcing element is considered in detail. The advantage of a discrete representation is that detailed information is directly obtained about the interaction of the soil and the reinforcing members (e.g. bound stresses, stress concentrations, edge effects etc.). The chief disadvantage is excessive computational cost for real structures containing large numbers of reinforcing elements.

A comparative study performed by Herrmann and Al Yassin (7) demonstrated that the two approaches yield very similar results and consequently they can be applied with equal accuracy to the analysis of reinforced soil systems. However, in general for large two-dimensional and three-dimensional configurations, only the composite approach is economically feasible.

While finite element computer programmes have opened up a possibility of obtaining improved solutions to the problem of reinforced earth, they are still not developed far enough to be used for design purposes. However the results obtained by finite element analyses can be translated into proper hand analysis methods (17).

As with the finite element analysis methods, there are two mechanisms involved in hand analysis methods : (a) composite behaviour and (b) dense tie-back behaviour.

The design procedure advocated by the French parent company Terre Armée is based on an assumed composite action between the granular soil backfill and the reinforcing ties (14, 20). It is accepted that if a failure wedge develops, it will occur in the unreinforced fill immediately behind the reinforced fill. Treating the reinforced fill as a single unit, the earth pressure distribution is calculated as if the reinforced fill was a rigid gravity retaining structure (fig. 2a).

The tensile force in a tie at any depth h below the surface of the reinforced zone is calculated assuming equilibrium with the total lateral earth pressure force acting on the tributary area of the wall pertaining to this tie :

$$T = K_a \cdot \sigma_v \, S_x \, S_y \qquad (1)$$

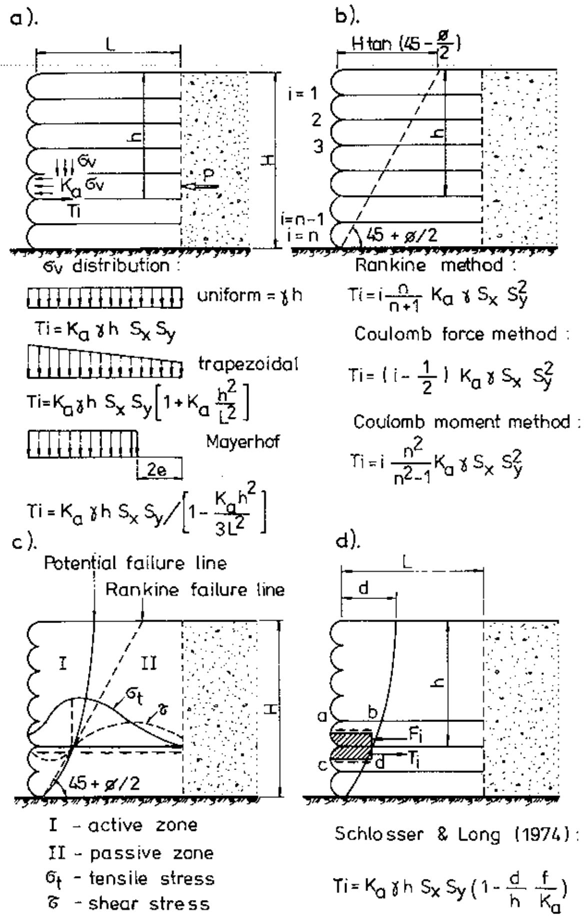


Fig. 2 - Methods of reinforcement tensile force evaluation.

where K_a is the active earth pressure coefficient, σ_v is the vertical pressure at depth h and S_x and S_y are the horizontal and vertical spacing of the ties. The vertical pressure is simply taken as overburden pressure, $\sigma_v = \gamma \cdot h$.

Schlosser and Long (15) have also considered the effect of the thrust from the retained fill on tensile forces in the reinforcement. This requires an assumption about the distribution of vertical stress and they have assumed a trapezoidal distribution and a Mayerhof distribution which depends on the eccentricity, e, of the reaction at the level considered, as shown in fig. 2a. When the reinforcements are sufficiently long the equations based on the trapezoidal and Mayerhof distribution can be replaced by the first equation given in fig. 2a, corresponding to an uniform distribution of the vertical stress.

Two types of inconsistency develop in the Terre Armee procedure, namely : (1) while the failure wedge is assumed to act behind the reinforced zone, the horizontal earth pressure within the reinforced soil is calculated using the plastic equilibrium coefficient K_a ; (2) while the procedure is based on composite material properties, calculations involving internal equilibrium of the reinforced zone are not performed. Based on considerations for the overall stability of the reinforced zone and on field observations, the tie length, L, is arbitrarily assumed to be not less than 0.8H for high walls and somewhat greater than H for low walls, where H is the total height of reinforced zone.

Additional design procedures have been developed (5, 9, 15) assuming that a plastic failure wedge develops immediately behind the face of the wall as is assumed in conventional retaining wall design. This would imply that the reinforced earth system acts as a densely reinforced tie-back system and not as a composite material. These procedures are based on either the Coulomb or Rankine methods of computing the active thrusts on the wall.

The Coulomb methods consider the overall force or moment equilibrium of the entire wall, whereas the Rankine method considers the equilibrium of a single element of soil at any depth. In the Coulomb method a triangular distribution of reinforcement force with depth is assumed whilst in the Rankine method it is assumed that the major principal stress acts in a vertical direction. The equations obtained by above mentioned methods for the tensile force in the strip element "i" are given in fig. 2b. For large values of i and large number of reinforcement beds, n, the three methods provide similar values of the tensile forces (18). With a good approximation the tensile force at depth h below the surface of the wall can be taken as :

$$T = K_A \gamma h \, S_x \, S_y \qquad (2)$$

which is equivalent with the equation provided by the Terre Armee procedure for an uniform σ_v distribution.

The equations derived by Rankine and Coulomb methods for tensile force in the reinforcement were based on the assumption of a failure plane inclined at 45 + Ø/2 degrees to the horizontal. This wedge of pressure is greater than the pressure wedge defined by measuring, in full scale experiments, the position of the points of maximum tension in the reinforcement strips (fig. 2c). The points of maximum tie tension lie on a parabolic failure surface that separates the reinforced mass into an active zone, where the shear stresses are directed towards the facing, the soil having the tendency to pull out the strips, and a passive or resistant zone, where the shear stresses are directed towards the interior, soil having the tendency to restrain the strips. Tests on models and calculations using the finite element method have shown that the shear stress exerted by the earth on the reinforcement does not, at a given point, have identical values on both faces of the reinforcement (15).

Using the above mechanisms, Schlosser and Long (15) have proposed a design procedure based on the equilibrium of a soil element located in the active zone, as shown in fig. 2d. The proposed analysis considers the soil element a b c d acted upon the tie tension, T, at the active zone boundary, the reaction, F, of the passive zone and the shear stresses on the faces ab and cd. An approximate procedure is used to derive the tie force equation given in fig. 2d.

In the design of a reinforced earth wall the problem of both internal and external stability must be considered.

INTERNAL STABILITY CONDITIONS

Internal stability of reinforced earth retaining walls depends on the stability and performance of both the reinforcing strips and the skin elements.

To insure adequate performance, the ties should not fail in tension or due to lack of frictional resistance under applied loads.

If the yield or failure stress of the tie material is σ_y, the width of the tie is b, and the thickness is t, then the factor of safety against failure or breaking in the ties, based on Eq. 2, is :

$$F_y = \frac{\sigma_y \, b \, t}{K_a \gamma h \, S_x S_y} \qquad (3)$$

For an uniform rectangular distribution of reinforcing elements of equal tensile strength, the lateral strains in the strips are likely to increase with depth. Thus near the top of the wall the soil between the strips will be close to an "at rest" condition whilst near the base of the wall the soil will approach the active failure condition (fig. 1b). On the other hand, model tests carried out by different authors (9, 10) appeared to partially substantiate the lateral earth pressure theory based on a K_a condition. As the elastic and strength properties of the test models are several orders of magnitude smaller than the field models, the observed failure mode within the test model reinforced earth may not actually take place in the field. Tie force measurements on full-scale tests (2, 5) suggest that an at rest K_o pressure distribution may by more appropriate for tie breaking analysis. Based on the preceding observations and on analytical finite element studies, Shen et al. (17) proposed the K_a coefficient in Eq. 3 to be replaced by the K_o coefficient.

Calculation for the factor of safety against slippage requires an estimation of pullout resistance of the ties. If the small amount of frictional resistance against the edges of the tie is neglected, the total frictional resistance of a tie at depth h may be considered as :

$$R_f = 2 \, \ell \, b \, \gamma h \, f \qquad (4)$$

in which ℓ is the effective length of the tie which resists the pullout force and f is the coefficient of friction between the reinforcement and the soil.

For a rectangular reinforced earth wall, the following equation for the factor of safety against slippage is obtained on the assumption $\ell = L$, the total length, that is the whole length of the reinforcement strip is involved in pulling out through the soil :

$$F_s = \frac{2 \, L \, b \, f}{K_a \, S_x \, S_y} \qquad (5)$$

However, a more realistic assumption would be that only the length of reinforcement extending beyond the theoretical failure line is active in preventing this mode of failure (9). As the factor of safety based on this assumption increases with depth, the top layer of reinforcement is the most critical and has a factor of safety given by the following equation :

$$F_s = \frac{2 \, b \, f \, [L - H \tan (45 - \emptyset/2)]}{K_a \, S_x \, S_y} \qquad (6)$$

where $\emptyset$ is the angle of shearing resistance of the soil.

Al-Hussaini and Perry (2) approximated the shear stress distribution, beyond the theoretical failure line, by a parabola and on this basis evaluated the total effective resisting force due to friction and corresponding factor of safety against pullout failure as :

$$F_s = \frac{4 \, b \, f \, [L - H \tan (45 - \emptyset/2)]}{3 \, K_a \, S_x \, S_y} \qquad (7)$$

Based on the finite element results and field performance, Shen et al. (17) consider that the stress state within the reinforced earth wall is approximately the K_o condition, whereas in the backfill just behind the wall approaches the K_a condition. It is also assumed that a linear force gradient develops in the reinforcing strips between the K_a state at the face, where they are bolted to the skin plate, and the K_o state in the interior. At the back edge of the strips the force must start from zero and build to its maximum value over a finite length. Thus, the total length of the strips must exceed the development length at both ends :

$$L \geqslant \ell_1 + \ell_2 + \Delta \qquad (8)$$

where :

$$\ell_1 = \frac{(K_o-K_a)\,S_x\,S_y}{2\,b\,f} \;(\text{face}) \quad ; \quad \ell_2 = \frac{K_o S_x S_y}{2\,b\,f} \;(\text{back})$$

and Δ = an assumed desirable distance of constant force in the interior of the wall.

Field pulling tests performed by Chang (5) using galvanized reinforcing steel strips, put into evidence typical load-deformation curves as shown in fig. 3a with yielding, peak, and residual load points clearly defined. The yielding load, representing the proportional limit of the load-deformation relationship, is the yield capacity of the steel or the maximum possible frictional grip of the compacted soil without the introduction of strain in the soil. The peak load represents the maximum mobile pulling resistance of the composite material of the reinforcement and soil. After the peak load, the strip becomes partially loose and progressively, the whole length of the strip starts sliding when the pulling loads drop to the residual level. If the strips break there will be no peak load or residual load.

Because the peak load represents the maximum mobilized friction grip, the pullout factors of safety were evaluated by Chang (5) using the peak loads as failure loads and the tensile strip loads from Eq. 2. The relationships between overburden height, h, strip length, L, and the factor of safety, F_s, which can be used as a guide for selecting the minimum length of reinforcement required for a given wall height, are given in fig.3b. Because the residual load, representing a complete slippage failure, is much lower than the peak load, a conservative factor of safety of 4.0 is recommended for design purposes.

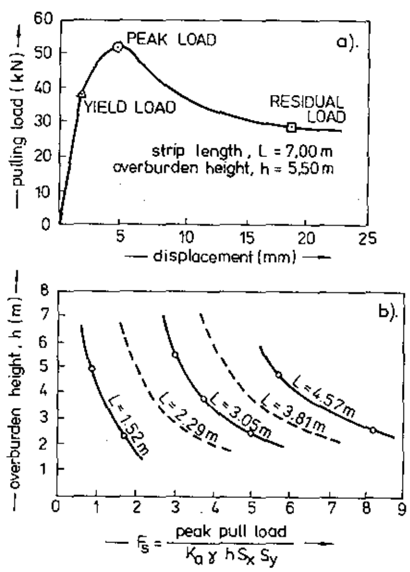
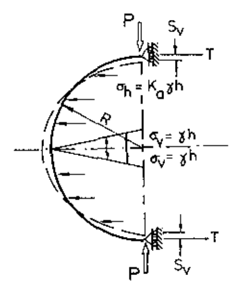


Fig. 3 - Typical load - deformation curve and experimental relationship for slipping factor of safety.

The skin plate which provides restraint for the soil between individual rows of reinforcing strips must be rigid enough to resist anticipated impact or shocks and flexible enough to tolerate a certain degree of deformation to conform with the settlement of the soil mass upon loading. The standard shape of the metal skin plate used by Terre Armee company consists of semi-eliptical element of 25 to 33 cm high with a thickness of about 3 mm.

Chang (5) developed a stress analysis of skin plates assuming a semicircular section of skin plate and hinged end conditions (fig. 4). The load P represents the resultant force, transfered from an uniform vertical pressure acting along an effective length of reinforcing strip, causing a vertical deformation, S_V. For design purposes, a value of S_V can be determined by estimating the structure settlement for one of the major design functions. Once the unknown load, P, is determined, the stresses developed in the skin plate can be readily calculated. Field performance studies have shown that the computed circumferential stresses based on this procedure agree reasonably well with the measured data.



$$P = \frac{4EI\,S_v}{\pi R^3} - \frac{4K_a\,\gamma h R}{3\pi} - \frac{\gamma h R}{24\pi}(3\pi - 32)$$

Fig. 4 - Loading diagram and design equation of skin plate.

EXTERNAL STABILITY CONDITIONS

The overall stability of a reinforced earth wall may be analysed by considering the reinforced earth mass as a solid block or a gravity type of concrete retaining wall. Resistance to bearing capacity, sliding at the base of the structure and overturning of a reinforced earth wall is investigated by Terre Armee procedure in the same manner as is done for design of retaining walls. Overturning due to lateral earth pressure behind wall is probably the least important and most dubious mode of failure. The analytical and field performance studies indicated that the shear effect at the back edge of the wall negated the overturning effect (17). However, if this shear could not be mobilized due to shear failure or time related relaxation, then this mode of failure may be applicable.

As in any retaining structure, the stability analysis should include calculations to identify the possibility of a general failure which would take out the reinforced earth mass along with a portion of the embankment. When failure surface intersects the reinforced earth structure, the internal resistance forces mobilized within the reinforced earth mass must be considered.

An interesting approach of this problem was recetly presented by Romstad et al. (12). The approach is basically a modification of conventional slope stability analysis incorporating the influence of the metalic reinforcement. The hypothesized failure surface, premised upon the finite element studies, consists of two straight lines with the transition occuring at the back edge of reinforcement (fig. 5). The strip resultants are assumed horizontal at failure. Solutions were developed in terms of two nondimensional design parameters, C_1 and C_2, as defined in fig. 5, permitting the rapid estimation of wall heights for a wide variety of parametric design conditions.

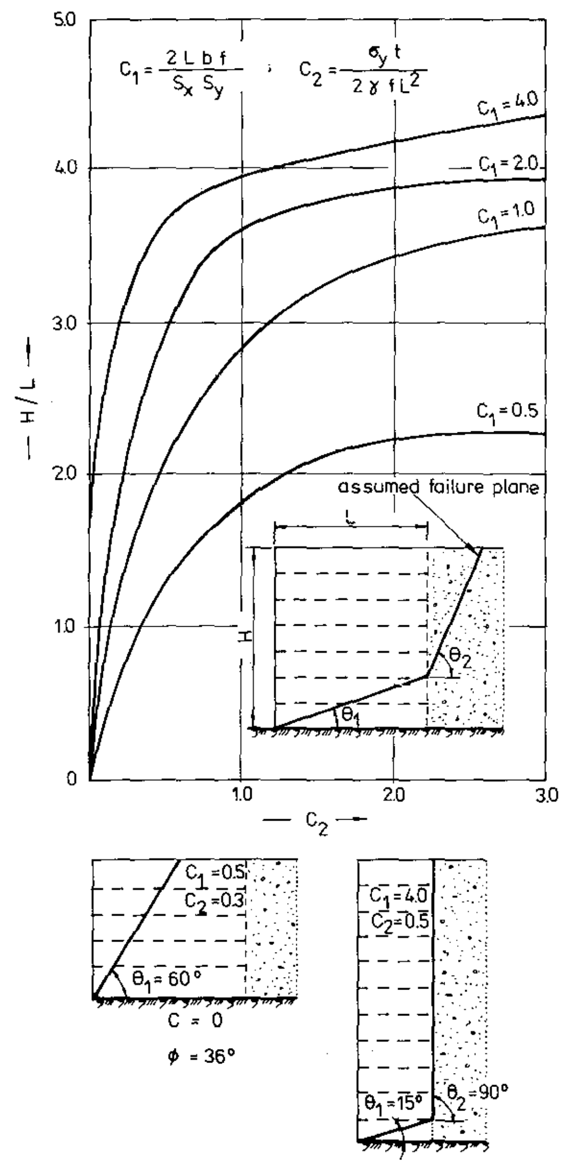


Fig. 5 - External stability analysis based on a general bilinear failure surface.

As observed in fig. 5, the value of H/L ratio increases with the increasing of both C_1 and C_2. An increase of C_1 is equivalent with decreasing of reinforcement horizontal and vertical spacing, while an increase of C_2 is equivalent with increasing the yield capacity of the strips, when all other parameters are hold constant. The solution for any given value of C_1 with increasing C_2 becomes horizontal when pullout becomes the mechanism of failure for all strips.

Generally, the larger values of C_1 and C_2 which represent greater percentages of reinforcement relative to the contributing area of soil, result in decrease the value of θ_1 and increase the value of θ_2. Normally, laboratory tests (9) are designed for small values of C_1 and/or C_2 because of low overburden stress, and hence they fail with the classical Rankine failure plane, while prototype systems (2) are designed with much larger values of C_1 and C_2, and thus the failure begins to represent merely a sliding of the entire reinforced earth mass as a monolitic unit. This is clearly illustrated in the lower part of fig. 5. Consequently, small scale models can only provide an approximation of the actual behaviour of the modeled prototype.

CONCLUDING REMARKS

In recent years reinforced earth has been widely used for construction of earth retaining structures.

The concept of reinforced earth is simple, but the mathematical prediction of its behaviour is complex. As a result, considerable experimental and theoretical work was done to improve analysis, design, and construction methods of this system.

Numerous, both hand calculation and computer finite element methods are now available for internal and external stability analysis of a reinforced earth wall. The paper gives a short comparative review of these methods. The method validity is checked by theoretical considerations and by comparison with data from experimental model tests. Because of the difficulties in exact scaling in laboratory model tests, it is concluded that a reasonable way of providing a realistic approach for designing reinforced earth walls is to study the performance of large-scale models in the field.

It appears that many complexities and unknowns still remain before reinforced earth walls may be designed and constructed using smaller factors of safety.

REFERENCES

1. AL-HUSSAINI M., JOHNSON L.D., (1977), Finite element analysis of a reinforced earth wall, Techn. Rep. S-77, U.S. Army Eng. Waterways Station, Vicksburg, Miss.

2. AL-HUSSAINI M., PERRY E.B., (1978), Field experiment of reinforced earth wall, J. Geot. Eng. Div., ASCE, vol. 104, N°GT3.

3. ANTONESCU I., (1974), Reinforced earth, Sinthesis Studies, N° 3, Center of Hydrotechnical Information and Documentation (in Romanian).

4. BALLY R.J., RODEANU C., (1978), Use of reinforced earth in hydrotechnical construction, CNST Symposium on Reinforced Earth (in Romanian).

5. CHANG J.C., (1974), Earthwork reinforcement techniques, Res. Report, Transportation Lab., Calif., Dept. of Transportation.

6. CHANG J.C., FORSYTH R.A., (1977), Finite element analysis of reinforced earth wall, J. Geot. Eng. Div., ASCE, vol. 103, N°GT7.

7. HERRMANN L.R., AL YASSIN Z., (1978), Numerical analysis of reinforced soil systems, Symposium on Earth Reinforcement, ASCE Annual National Convention, Pittsburg.

8. KASSIFF G., KAPELOVITZ A., (1968), Strength properties of soil-root systems, Technion Report, Haifa, Israel.

9. LEE K.L., ADAMS B.D., VAGNERON J.J., (1973), Reinforced earth retaining walls, J. Soil Mech. Found. Div., ASCE, vol. 99,

10. RICHARDSON G.N., (1976), The seismic design of reinforced earth walls, Ph. D. Dissertation, University of California, Los Angeles.

11. ROMSTAD K.M., HERRMANN L.R., SHEN C.K., (1976), Integrated study of reinforced earth - I, Theoretical formulation, J. Geot. Eng. Div., ASCE, vol. 102, N° GT5.

12. ROMSTAD K.M. et al., (1978), Stability analysis of reinforced earth retaining structures, Symposium on Earth Reinforcement, ASCE, Annual National Convention, Pittsburg.

13. SANGLERAT G., (1971), Massifs de terre armée, Revue Technica.

14. SCHLOSSER F., VIDAL H., (1969), La terre armée, Bull. liaison Lab. P. et Ch. N° 41.

15. SCHLOSSER F., LONG N.T., (1974), Recent results in French research on reinforced earth, J. Constr. Div., ASCE, vol. 100, N° C03.

16. SCHLOSSER F., (1977), La méthode des éléments finis appliquée aux ouvrages en terre armée, Bull. liaison Lab. P. et Ch., N° 63.

17. SHEN C.K., ROMSTAD K.M., HERRMANN L.R., (1976), Integrated study of reinforced earth - II : Behaviour and design, J. Geot. Eng. Div., ASCE, vol. 102, N° GT6.

18. SYMONS I.F., (1973), Reinforced earth retaining walls, Highways and Road Construction, N° 4.

19. UESAWA H. et al., (1969), A large scale experiment on reinforced earth, Publ. Japanese Nat. Railways Res. Inst. (in Japanese).

20. VIDAL H., (1966), La terre armée, Ann. ITBTP, série M/30.

21. WESTERGAARD H.M., (1938), A problem of Elasticity suggested by a problem in Soil Mechanics, Mechanics of Solids, Timoshenko 60th Anniversary Vol. Macmillan & Co, N.Y.