

Stability analysis of reinforced slopes based on apparent cohesion method

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ABSTRACT: This paper presents stability analysis of reinforced slopes. Reinforced earth is treated as a soil with additional anisotropic „apparent” cohesion. This „reinforced effect” depends on the soil and reinforcement properties and the inclination of failure surface at its intersection with the reinforcement as well. The tensile rupture and pull-out rupture of reinforcement are taken into account.

This approach based on simplified Bishop method is adapted to analysing reinforced earth constructions. The modification permits to assess the internal and overall stability of reinforced slope. It also enables to check the correctness of design considerations i.e. provision and distribution of reinforcement in construction.

The theoretical prediction of external stability is compared with full scale test results. The overall stability analysis of soil nailing structure at the toe of steep slope of cliff coast is also presented.

1. INTRODUCTION

In reinforced earth structures the reinforcement carries axial forces which increase the shear resistance of the soil and improve stability. The additional strength imparted by the reinforcement could be represent by an apparent cohesion. So called „cohesion theory” was proposed by Schlosser & Long (1973) and Hausmann (1976). The allowable of such a purpose is confirmed by the results of triaxial compression tests.

The aim of this study is to present the simple application of „apparent cohesion” approach to the analysis of the internal and overall stability of reinforced slope. In the paper the case of uniform and horizontally reinforced soil is considered. Reinforced earth is treated as a matrix soil with an additional anisotropic cohesion. To determine an amount of „reinforced cohesion” the reinforced soil mass is homogenised first, and the equilibrium of the forces acting across the slip line is examined. Subsequently, the classical slope stability analysis is used as a technique of solution.

2. HOMOGENISATION AND FAILURE MECHANISMS

In the present paper the soil reinforced with horizontally and uniformly placed inclusions is considered. Reinforcement, in the form of geotextile, geogrid, bars or strips is assumed to be flexible and withstand only the tension forces. As mentioned above, the soil-reinforcement system is represented as a soil with apparent cohesion c_R (Fig.1) and can be analysed using slightly modified conventional slope stability analysis procedure.

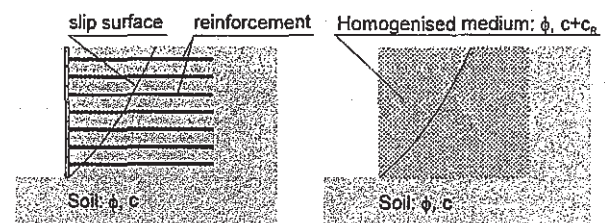


Fig.1 Homogenisation of reinforced soil

The safety factor is evaluating with respect to failure along potential circular sliding surfaces. The shearing resistance of the soil, as defined by Coulomb-Mohr's failure criterion, is entirely mobilised all along the failure surface. At failure, also the tension rupture or pull-out of reinforcement appear at the sliding surfaces developed in the soil.

The strengthened behaviour of reinforcement is described by the coefficient σ_0 , which indicates the tensile strength of reinforcement per unit cross section of entire structure

$$\sigma_0 = \frac{n \cdot T}{A_c}, \quad (1)$$

where T is a tensile limit force for single element of reinforcement, A_c is a cross section area of construction normal to the reinforcement direction and n is the number of reinforcement elements. For tensile rupture $\sigma_0 = \sigma_0^t = \text{const.}$, and can be defined for sheets of geotextile or geogrid as

$$\sigma_0^t = \frac{T_G}{\Delta H}, \quad (2)$$

where T_G is limit force per unit length of reinforcement and ΔH denotes the vertical spacing. For reinforcing bar or strips, σ_0 can be taken as

$$\sigma_0^t = \frac{T_B}{\Delta H \cdot \Delta V}, \quad (3)$$

where T_B is a tensile limit force of a single bar or strip and ΔH and ΔV denote horizontal and vertical spacing, respectively.

Besides of tensile rupture it is necessary to assess the pull-out resistance zone in reinforced mass (Fig.2).

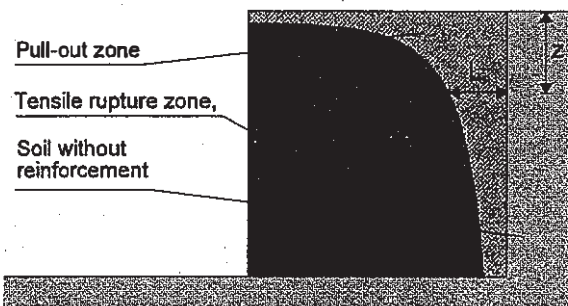


Fig.2 Tensile rupture zone and pull-out zone in the homogenised structure

This zone is located close to the end of reinforcement and its length L_p at a depth z can be calculated as

$$L_p = \frac{T}{2 \cdot w \cdot (\mu \cdot [z \cdot \gamma + q] + \psi \cdot c)}, \quad (4)$$

where $T=T_G$ for sheet reinforcement or $T=T_B$ for bar or strip, w is the width of reinforcement per metre run of the slope (for full width of the slope $w=1$), μ is the frictional bond coefficient, q is the surcharge on top of the slope, γ is the unit weight of the soil, ψc describes soil-reinforcement adhesion for fills exhibiting a cohesion c .

For pull-out rupture $\sigma_0 = \sigma_0^p$, where $\sigma_0^p = \chi \cdot \sigma_0^t$. A coefficient χ , which indicate the mobilisation of reinforcement strength range of χ is restricted to $0 \leq \chi \leq 1$. It is assumed that σ_0^p is equal to 0 at the end of reinforcement, and increases steadily to a maximum value $\sigma_0^p = \sigma_0^t$ at border between the pull-out and the tensile rupture zones. Hence, a coefficient χ may be written as

$$\chi = \frac{L}{L_p}, \quad (5)$$

where L is the distance from the end of reinforcement (Fig.3).

3. APPARENT COHESION AS AN EFFECT OF REINFORCEMENT IN SOIL

The force equilibrium method assuming the bilinear sliding surface is proposed to determine the algebraic formula for apparent cohesion. This approach is based on the simple limit analysis described by Jewell et al. (1980) for direct shear test.

Figure 4 illustrates failure surface passing through the reinforcing mass. N and S denote the normal and shear forces, R is the soil reaction force, ϕ is an

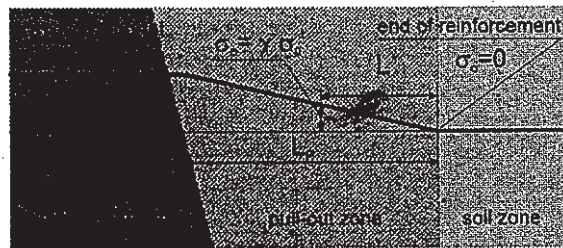


Fig.3 Dispersion of coefficient σ_0 longwise the layer of reinforcement

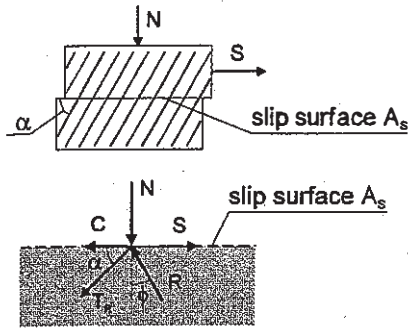


Figure 4. Force equilibrium analysis in the direct shear apparatus

angle of internal friction, C is the cohesion resistance force and T_R is the resultant tensile reinforcement force. The direction of reinforcement is inclined at α to the slip surface.

The analysis of the equilibrium of forces gives the following expression

$$S = N \cdot \tan \phi + C + T_R \cdot (\sin \alpha \cdot \tan \phi + \cos \alpha). \quad (6)$$

This formula may be presented in terms of stresses:

$$\tau = \sigma \cdot \tan \phi + c + \frac{T_R}{A_s} \cdot (\sin \alpha \cdot \tan \phi + \cos \alpha), \quad (7)$$

where τ , σ and c denote shear stress, normal stress and cohesion, respectively, and A_s is a slice failure surface area.

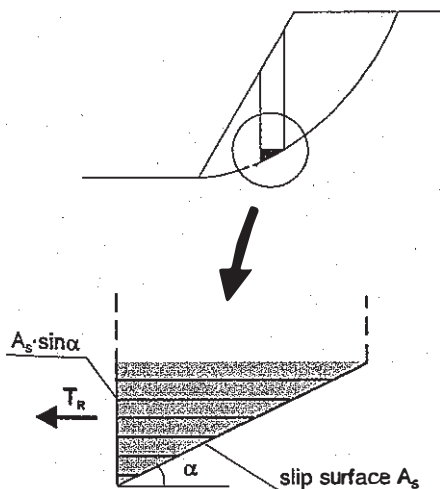


Fig.5 Relationship between coefficient σ_0 and resultant of tensile forces in reinforcement

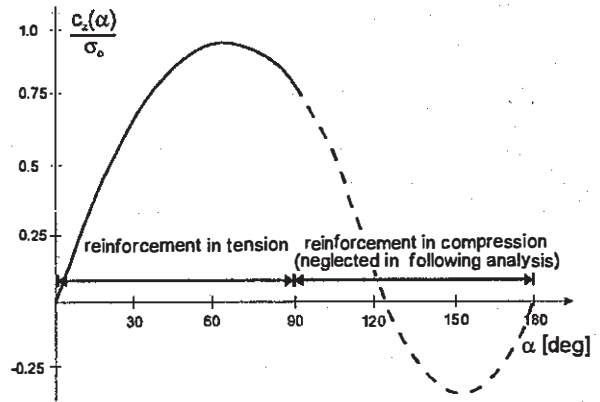


Fig.6 Increase of apparent cohesion against reinforcement orientation for $\sigma_0 = \text{const.}$ and $\phi = 34^\circ$

The comparison of expression (1) and the Figure 5, shows, that the value of σ_0 is related to the resultant of tensile forces T_R by

$$\sigma_0 = \frac{T_R}{A_s \cdot \sin \alpha}. \quad (8)$$

Equations (7) and (8) may be combined to give the expressions

$$\tau = \sigma \cdot \tan \phi + c + c_R(\alpha, \sigma_0, \phi), \quad (9)$$

where c_R is an additional anisotropic cohesion increasing the shearing resistance of a soil

$$c_R = \sigma_0 \cdot \left(\sin^2 \alpha \cdot \tan \phi + \frac{1}{2} \cdot \sin 2\alpha \right). \quad (10)$$

In tension rupture zone the coefficient $\sigma_0 = \text{const.}$ and the value of c_R depends only on angle α . In Figure 6 such a relationship is plotted against reinforcement orientation.

4. COMPARISON WITH THE DIRECT SHEAR TESTS RESULTS

In order to assess the accuracy of the proposed formula the theoretical prediction is compared to the results of direct shear tests on reinforced sand (Gray et al. (1983)) and (Jewell et al. (1987)).

The tests described by Gray et al. (1983) were carried out using sand reinforced with reed fibres. The measured friction angle of sand was $\phi = 39^\circ$, and the cross-sectional area of apparatus was $A_s = 0.0335 \text{ m}^2$. Fibre cross-sectional area A_R and tensile strength

T_B were as follow: $2.54 \times 10^{-6} \text{ m}^2$ and 33485 kN/m^2 . In one series of the tests the influence of fibres inclination (α) on the shear strength increase (c_R) was measured. In Table 1. the test results were compared with the theoretical prediction.

Table 1. Results of direct shear tests on sand reinforced with 6 reed fibres (after Gray et al. (1983))

| α [deg] | c_R [kN/m ²] | c_R^{exp} [kN/m ²] |
|----------------|----------------------------|---|
| 30 | 9.7 | 9.6 |
| 60 | 15.9 | 16.7 |
| 90 | 12.4 | 11.9 |

The tests presented by Jewell et al. (1980) were carried out using sand sample reinforced with rough elastic bars, placed at an angle α to the horizontal plane. In three of the tests the additional shearing resistance (c_R) and the maximum reinforcement force (T_R) were measured. The internal friction angle of sand $\phi=54.4^\circ$, and the area in central plane in the direct shear apparatus $A=0.0386\text{m}^2$. The number of bars $n=6$. The reinforcement force T_R in one bar, angle of reinforcement from the horizontal direction α_i , are listed in Table 2. In this Table the values of the analytical and experimental apparent cohesion, c_R and c_R^{exp} , are also compared.

Table 2. Results of direct shear tests on reinforced sand (after Jewell et al. (1987))

| Test | T_R [N] | α [deg] | c_R [kN/m ²] | c_R^{exp} [kN/m ²] |
|------|-----------|----------------|----------------------------|---|
| S9Y | 75 | 64 | 19.7 | 16.0 |
| S8Y | 25 | 65 | 6.6 | 7.5 |
| S7Y | 10 | 66.5 | 2.6 | 2.5 |

The comparison with the direct shear tests show that the proposed method leads reasonable prediction.

5. MODIFICATION OF THE BISHOP METHOD

Circular slip analysis, based on simplified Bishop method, can be modified to assess the stability of reinforced slope (Fig.7). The active zone, above the slip surfaces is divide into n vertical slices. The weight and width of i th slice are W_i and b_i , respectively. The tangent to the slip surface at the i th

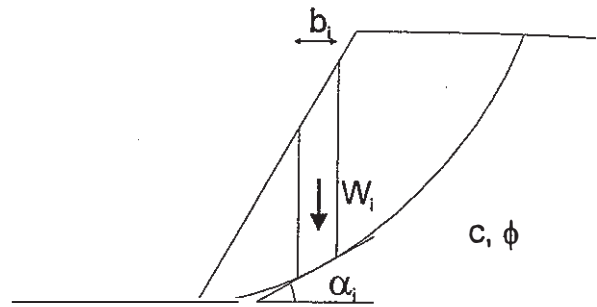


Fig.7 Circular slip analysis in simplified Bishop method

slice is inclined at α_i to the horizontal. When used for the assessment of unreinforced and unsaturated slope the simplified Bishop method takes the form:

$$F = \frac{1}{\sum_{i=1}^n W_i \cdot \sin \alpha_i} \cdot \sum_{i=1}^n \frac{c \cdot b_i + W_i \cdot \text{tg} \phi}{\cos \alpha_i \left(1 + \frac{\tan \phi \cdot \tan \alpha_i}{F} \right)} \quad (11)$$

Apparent cohesion approach required only the slight modification of this procedure. For reinforced soil, the cohesion c is not constant, because the additional anisotropic cohesion c_R , given by expression (10) is incorporated into the programme. Thus, for i th slice the cohesion c_i is given by the expression:

$$c_i = c + c_R^i(\alpha_i, \sigma_0) \quad (12)$$

This method is used to modify a standard computer code, to assess the stability of reinforced slope. In additional subroutine the value of σ_0^t is calculated. Then the length of a pull-out zone L_p , the distance between the middle of the slice and the end of reinforcement L , and coefficient χ , are determined for each slice. In unreinforced zone both χ and σ_0 are equal to 0. In pull-out zone the value of σ_0 is equal to σ_0^p and can be calculated over the range $0 \leq \chi \leq 1$ from the expression $\sigma_0^p = \chi \cdot \sigma_0^t$. In tensile rupture zone χ is equal to 1, and $\sigma_0 = \sigma_0^t = \text{const}$. When the value of σ_0 has been determined, the corresponding value of apparent cohesion may be obtained from (10). The further calculations are performed using standard procedures of simplify Bishop method.

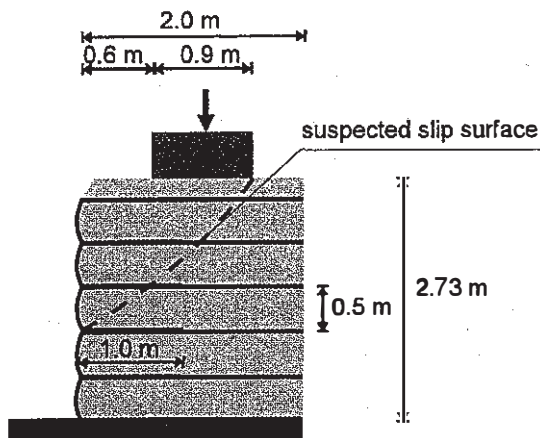


Fig.8 Cross-section of the full scale test abutment (after Thamm et al. (1990))

6. COMPARISON WITH THE FULL SCALE TEST

The full scale test on the geotextile reinforced abutment was achieved by Thamm et al. (1990) in Bundesanstalt für Strassenwesen in Germany. A comparison between the prediction made using the proposed apparent cohesion method and the results of this tests has been carried out in order to assess the accuracy of the methodology.

The test was conducted on a 2.73 m high structure reinforced with five layers of non woven needle-punched geotextile (Fig.8). The gravely sand used as a fill material was compacted to obtain a density of 19.3 kN/m^3 . The value of a friction angle and cohesion were 39° and 8 kPa , respectively. The 2 m length of reinforcement was placed in five layers at a vertical spacing $\Delta H = 0.5 \text{ m}$. Tensile strength tests were conducted on reinforcement and yielded the approximate strength of 17.5 kN/m . The structure was loaded continuously with a concrete slab (2.4 m long and 0.9 m wide) and a loading frame to the failure. The failure was registered after a load 640 kN as a linear crack developed at about 0.3 m behind the back of the slab.

The numerical code based on apparent cohesion approach was used to assess the internal stability of previously described reinforced structure. In the first step the circular slip failure line corresponding to minimum safety factor F was found. Subsequently, the calculation for the different value of the external loading was repeated, so that the safety factor would be equal to 1.0. The calculated pull-out zone, and the

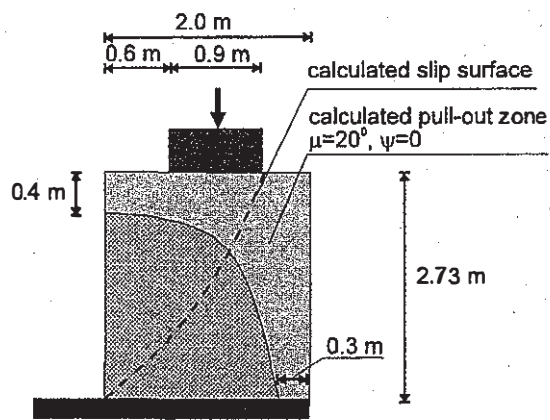


Fig.9 Pull-out zone and slip surface calculated using modified Bishop method

analytical slip surface are presented in Fig. 9. The value of assessed external load was 648 kN.

7. STABILITY ANALYSIS OF SOIL NAILING STEEP SLOPE OF CLIFF COAST

Circular slip analysis may be employed to assess overall stability involving slip surfaces passing wholly or partly through the reinforced mass, the foundation soil and the fill behind the reinforced mass. To illustrate this application the analysis of soil nailing structure proposed to improve stability of Jastrzębia Góra cliff is presented.

Generalised geological cross-section of 30 m high cliff is shown in Fig.10. The base of cliff is a clay layer forming the lower part of slope up to 12 m above the sea level. Above that there are two horizons of moraine silty-clays separated by the sand.

The extensive sea abrasion of cliff's threshold cause deep landslides which usually reach the top entering the land up to several meters. It causes a real danger for infrastructure existing on the top of the cliff.

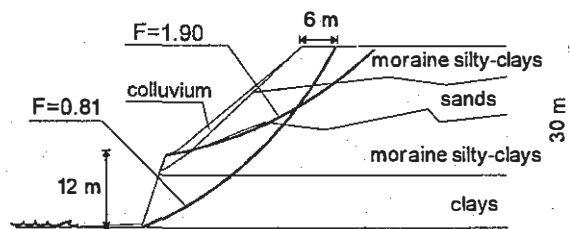


Fig 10. Generalised geological cross-section of Jastrzębia Góra cliff (after Tejchman et al. (1995))

The gabion's wall was proposed for protection of the toe of the cliff against the abrasion. However, such a construction did not improve the overall stability of slope (Fig.10). The whole slope was unstable ($F_{min}=0.81$), although the factor of safety F obtained for the upper part of the cliff was about 1.90.

The additional soil-nailed construction was proposed for improving the cliff stability (Fig.11). The design follows calculation method based on the theory of composites.

In the 12 m high structure, the nails, comprised length of 15 m, and a diameter of 32 mm, were set into the ground horizontally with 2 m vertical and horizontal spacing. Thin reinforced soil wall, similar to the Textomur system (Rüegger R. (1986)), was applied as a facing of soil-nailed construction.

For slope with an improvement, the overall stability reanalyses was performed using computer code based on apparent cohesion method (Fig.12), which confirmed the correctness of design considerations. It was shown that soil-nailed structure prevented

further slippage of the slope outwards and stabilised the cliff, raising the safety factor to 1.57.

8 CONCLUSION

The simple application of „apparent cohesion” approach to the stability analysis of reinforced slope is described in the paper. The algebraic formula, based on force equilibrium method is derived. This approach required only the slight modification of slope stability analysis procedure, to assess the internal and overall stability of reinforced slope. The comparison with the direct shear tests, and full scale test show that the proposed method leads reasonable prediction.

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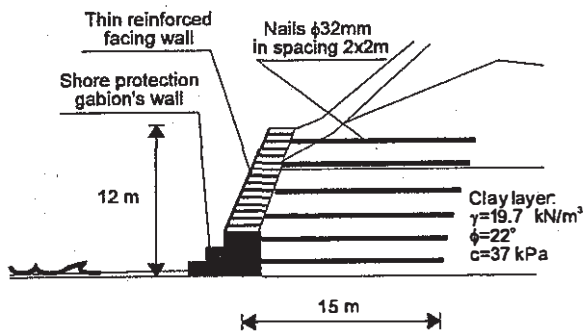


Fig. 11 The cross-section of proposed construction

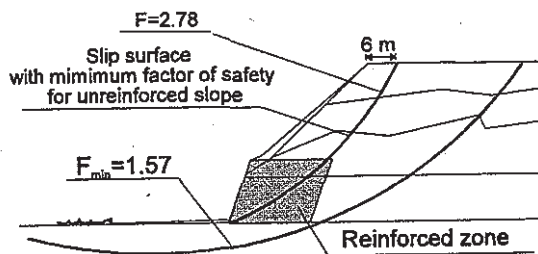


Fig. 12 Overall stability of soil-nailed cliff slope