

## Stability of a geosynthetically-reinforced embankment

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**ABSTRACT:** This paper presents an analytical method to evaluate the stability of a geosynthetically-reinforced embankment. The reinforcing inclusion is modeled as a layer of strengthened soil with anisotropic properties. The failure surface of the embankment against the sliding failure is determined using Mohr's circles of failure stresses at either the active or passive states. The developed procedure is used to evaluate the inclusion influence to the embankment stability.

### 1 INTRODUCTION

Geosynthetics have been used as reinforcing material to improve soil behavior in the geotechnical engineering works. Layers of geosynthetic sheets are placed horizontally in the embankment to increase the stability of the slope against slip failure. The reinforcing mechanism is provided by the shear force transformation in the contact between the geosynthetic and the soil particles. The vertical stresses in the embankment induce lateral strain of soil mass, which in turn mobilize tensile strain in the adjacent reinforcing inclusion. The strained inclusion provides additional lateral constraint to the adjacent soil through the development of shear stress.

Because the transfer mechanism between the reinforcing inclusion and the surrounding soil are adhesive and frictional forces, the constraint effect provided by the inclusion propagates a limited distance into the soil matrix. The reinforcing inclusion is modeled as a layer of homogenous strengthened soil. However, because of the extension resistant nature of most geosynthetic materials, this strengthened soil layer is modeled as a material having anisotropic and only tension resistant properties. This method relies on the macroscopic strength conditions of reinforced earth as generally stated by de Buhan et al. (1989).

### 2 FORMULATION

Based on the concept proposed by de Buhan et al. (1989), reinforced soil is formulated as a locally homogeneous anisotropic material. The reinforcing effect of a thin inclusion was assumed to be uniformly distributed into the vicinity of the soil mass, as shown in Figure 1. The horizontal normal stress of the reinforced composite unit  $\sigma_{xx}$  can be expressed as:

$$\sigma_{xx} = \sigma_{xx}^s + \frac{R_t}{\Delta H} \quad (1)$$

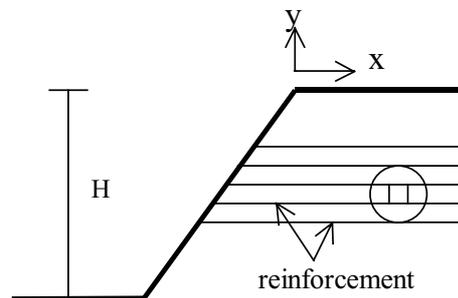
The failure envelope and slip surface directions of the reinforced composite unit are different from a pure soil mass. The principal stresses and failure envelope of the reinforced composite unit are presented as follows:

#### 2.1 The failure envelope of homogeneous anisotropic materials

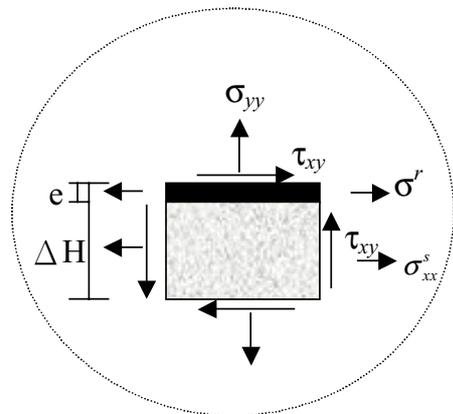
For a reinforced soil whose normal of the slip surface forms an  $\alpha$  angle with the inclusion (Figure 2), the normal and shear stresses resisted by the soil mass are:

$$\sigma_n^s = \sigma_n - \sigma_{ave}^r \cos^2 \alpha \quad (2)$$

$$\tau_n^s = \tau_n + \sigma_{ave}^r \sin \alpha \cos \alpha \quad (3)$$



(a) Reinforced slope



(b) Stresses of a reinforced composite unit

Figure 1. Schematic diagram of reinforced slope and reinforced composite unit (after de Buhan et al., 1989)

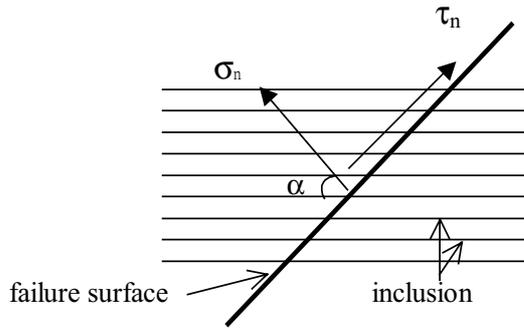


Figure 2. Direction of reinforced inclusion

According to the Mohr-Coulomb yield criteria, the soil stresses at failure satisfy the following inequality:

$$|\tau_n^s| \leq \sigma_n^s \tan \phi \quad (4)$$

Substituting Equation 2 into Equation 3 yields:

$$|\tau_n + \sigma_{ave}^r \sin \alpha \cos \alpha| \leq (\sigma_n - \sigma_{ave}^r \cos^2 \alpha) \tan \phi \quad (5)$$

Envelopes of the yield stresses are changed from OC and OG of the natural cohesionless soil into the dark solid lines AQOG of the reinforced composite shown in Figure 3.

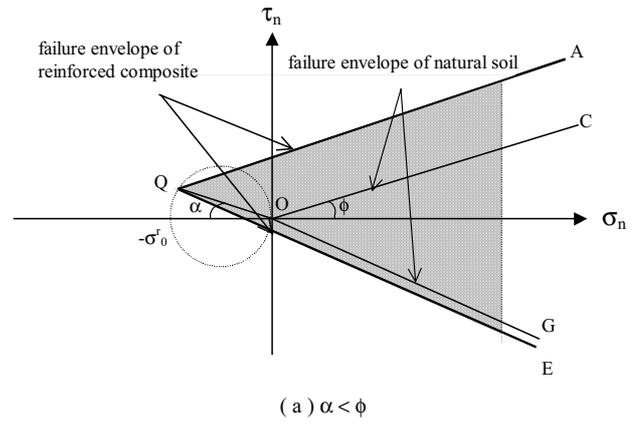
### 2.1.1 Reinforced composite unit under the crest or base

Because the reinforcement deployment alters the yield stress envelopes, the directions of the failure surface of the reinforced composite unit, as defined from stress Mohr's circle, and the yield envelope may be changed accordingly. The Mohr's circles of critical stress for a reinforced composite unit under the crest or base are shown in Figure 4. The pole is at point P. Stresses  $\sigma_A$  and  $\tau_A$  represented as point A in the stress circle are the normal and shear stresses of the failure plane having an angle  $\alpha$  with the horizontal plane. Because point A is on the yield verge so that angle  $\alpha$  can be determined, this angle coincides with that of the unreinforced soil mass,  $45^\circ + \phi/2$ . It can be concluded that the failure surfaces of the active state are the same for both reinforced and unreinforced soil.

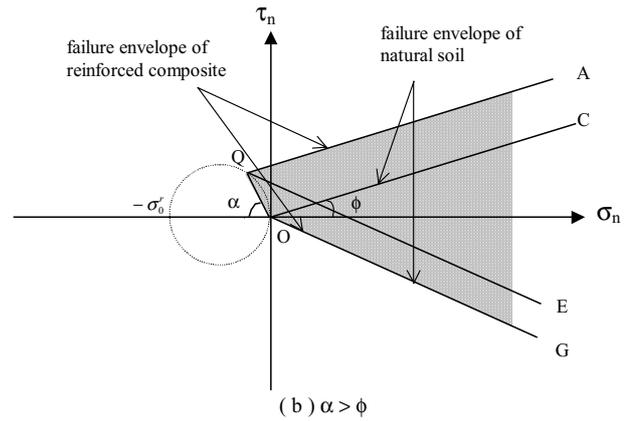
The passive state of the reinforced composite unit under a crest or base is shown in Figure 5. The soil mass on the right side of the failure surface squeezes that on the left side. The angle  $\alpha$  ranges between  $0^\circ$  and  $-90^\circ$ , and the relative deformation on both sides of the failure surface indicates that the reinforcement is subjected to compression. The reinforcement will not develop any resistance at this state because the flexible reinforcement resists tensile force only. Gray and Al-Refeai (1986) and Jewell (1980) investigated this phenomenon experimentally and concluded that a reinforcement contributes least when the angle of the failure plane and reinforcement orientation ranged between  $90^\circ$  and  $180^\circ$ .

### 2.1.2 Active and passive states of reinforced composite unit under slope

The maximum and minimum stresses for a reinforced composite unit under a slope are not in the vertical and horizontal directions as under a crest or base. The determinations of failure surfaces and critical stresses defined by these two extreme stresses are more complicated. According to the stability analysis by Rankine's failure plane, the active stress plane (line PA) forms an  $\alpha$  angle of  $45^\circ + \phi/2 - a$  with the horizon, as shown in Figure 6. Angle  $a$  is functions of the slope angle  $\beta$  and the principal stresses, and given as:



(a)  $\alpha < \phi$



(b)  $\alpha > \phi$

Figure 3. Failure envelopes of natural and reinforced composite

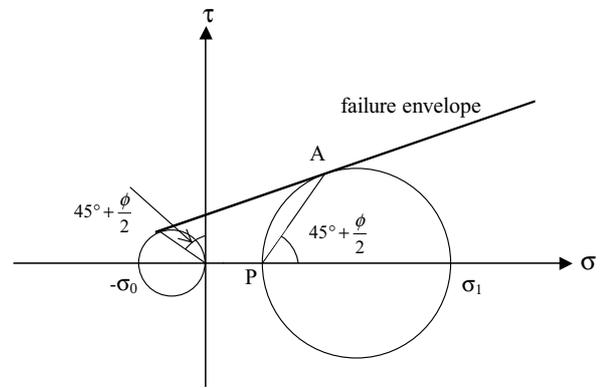


Figure 4. Stress Mohr's circle for reinforced composite unit under crest or base of slope

$$a = \frac{\varpi}{2} - \frac{\beta}{2} \quad (6)$$

$$\text{where } \varpi = \sin^{-1} \left( \frac{\sigma_1 + \sigma_3}{\sigma_1 - \sigma_3} \sin \beta \right)$$

By substituting  $\alpha$  angle into Equation 5, the failure envelope for the reinforced composite unit can be obtained through an iteration procedure and the result shown in Figure 6. In the passive state the angle between the failure surface and the

horizon is indicated as  $\theta$  in Figure 7. The angle  $\theta$  is also determined from an iterative calculation.

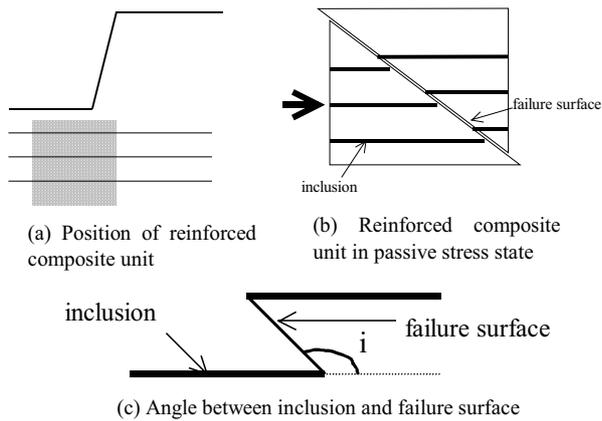


Figure 5. Reinforced composite unit in passive stress state

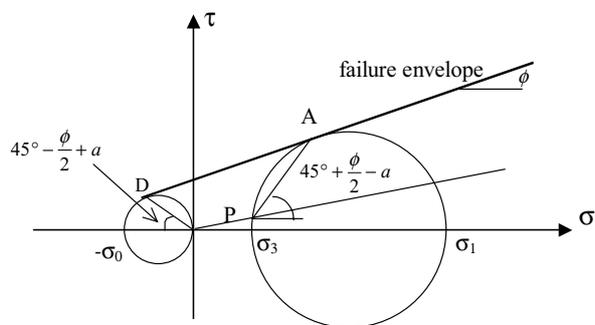


Figure 6. Stress Mohr's circle of reinforced composite unit under slope

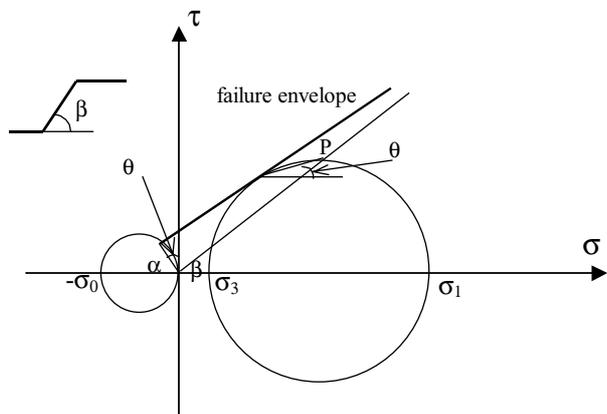


Figure 7. Passive state for reinforced composite unit under slope

### 3 ANALYTICAL RESULTS

#### 3.1 Alternation of the failure surface directions

As the stress state of the soil under a slope is represented by point Q in stress Mohr's circle; the direction of the failure surface for an unreinforced slope forms an angle  $\alpha$  with the horizon in the active failure state. The direction  $P_2S$  that makes an angle  $\theta$  with the horizon is the failure direction of the reinforced soil (Figure 8). Because the value of  $\theta$  is greater than

$\alpha$ , the failure direction becomes steeper for the reinforced soil, as indicated in section B of Figure 10. However, the failure surface becomes gentle for a passive state as indicated in Figure 9, where angles  $\alpha$  and  $\theta$  are the directions of the failure surface with the horizontal. A trace of the failure surface is shown in Figure 10 as portion A.

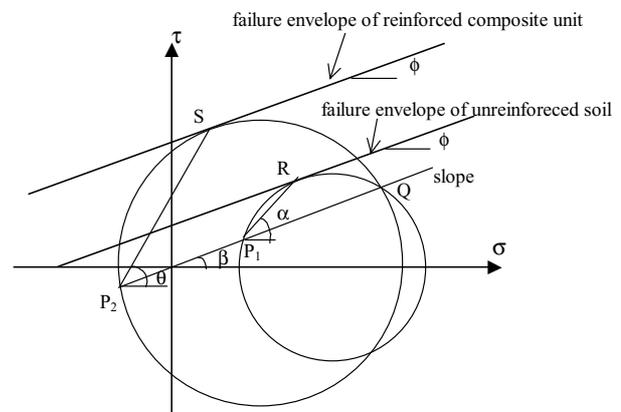


Figure 8. Direction change of reinforced composite unit at active state (under slope)

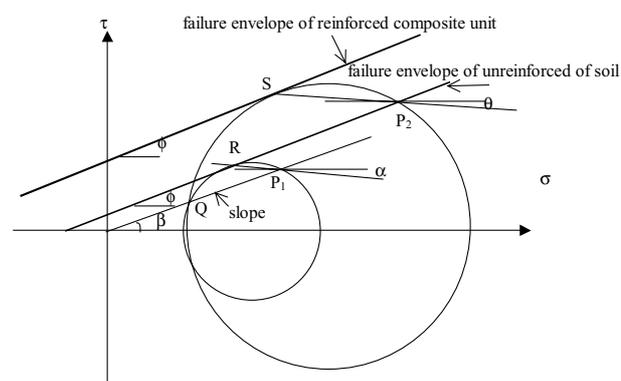


Figure 9. Direction change of reinforced composite unit at passive state (under slope)

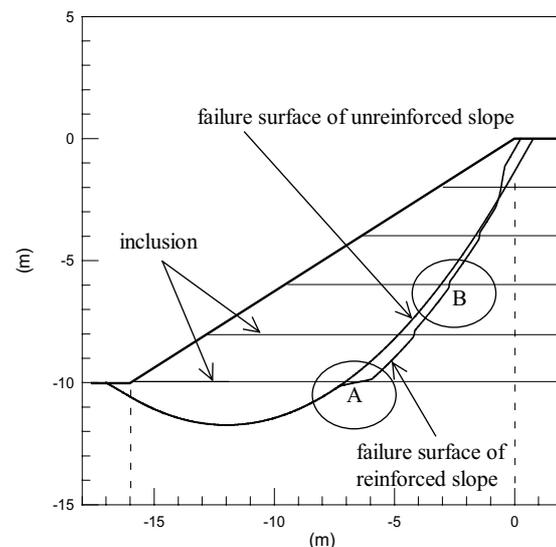


Figure 10. Failure surface change of a reinforced slope

### 3.2 Incipient point of the failure surface and the base altered by the reinforcing inclusion

The most possible failure surfaces for reinforced and unreinforced slopes have different incipient points with the base. The most possible failure surface is defined as a failure surface that corresponds to a relative minimum safety factor against slip failure. The incipient point is further away from the toe as the number of reinforcement layers is increased. The failure surface thus passes through more areas of soil and develops greater resistance to slip. Figure 11 shows the relationship between the safety factor against failure and the position of the incipient point for a reinforced slope. As the lengths of the reinforcement are assumed to be long enough so that tensile failure occurs, the relationship between the number of reinforcements and the incipient point position for a slope example is shown in Figure 12.

### 3.3 Influence of the stability from the reinforcement embedded length

Because the shear force is the transformation mechanism between the inclusion and soil particles, a slip or pullout failure occurs when the frictional force between the inclusion and soil in the portion extruding into the stationary part of the soil is less than the tensile force of the inclusion. The length of the inclusion changes the failure surface of the reinforced embankment. The inclusion length influence versus the safety factor corresponding to the various possible failure surfaces for a reinforced embankment is shown in Figure 13. As the incipient point of failure surface moves further away from the toe, the failure surface passes through a larger portion of the soil. Slopes with sufficient inclusion lengths will increase the safety factor, as indicated in Figure 13 for 7- or 10- meter inclusion lengths. For those slopes with shorter inclusion lengths (2- or 3- m inclusion length), the failure surface intercepts fewer inclusions and the safety factor is decreased as incipient point moves away from the toe. A decrease in the embedded length causes pullout failures because of insufficient anchor force.

## 4 CONCLUSIONS

The analytical results reveal that

- (1) The anisotropic property of the reinforced composite changes the failure envelope and failure direction compared to unreinforced soil. Safety factor and inclusion length estimation calculations can be determined accordingly.
- (2) As the lengths of the reinforcement are assumed to be long enough so that tensile failure occurs, the incipient point moves further away from the toe when the number of reinforcement layers is increased. The failure surface passes through more areas of soil and develops greater resistance to slip. However, the inclusion length that contributes resistance to slip reaches a limit, longer than a certain length will not increase the safety factor any further.

## 5 REFERENCES

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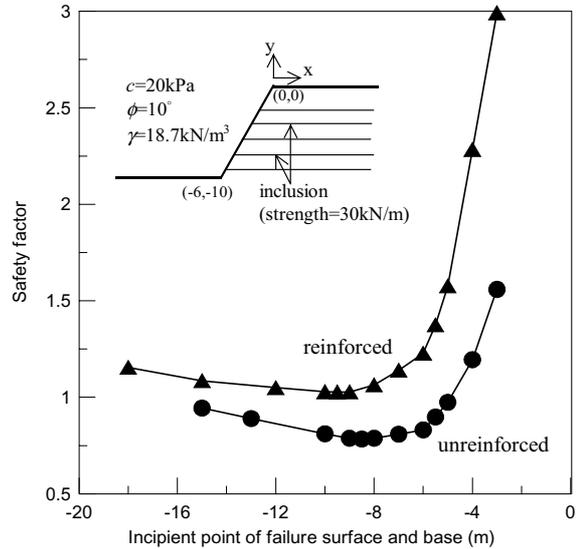


Figure 11. Safety factor vs. incipient point

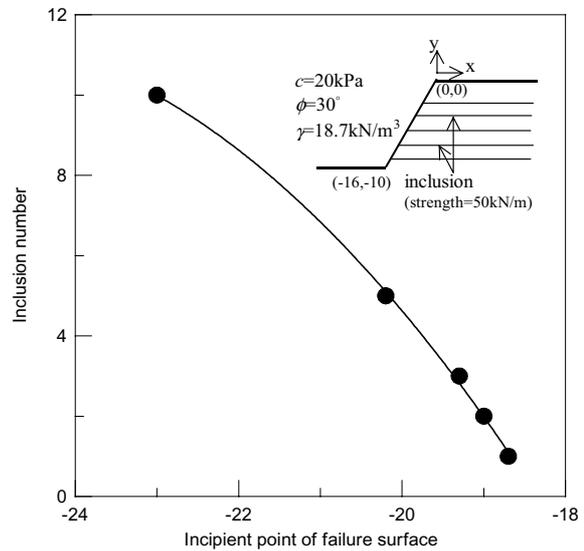


Figure 12. Incipient point of failure surface vs. inclusion number

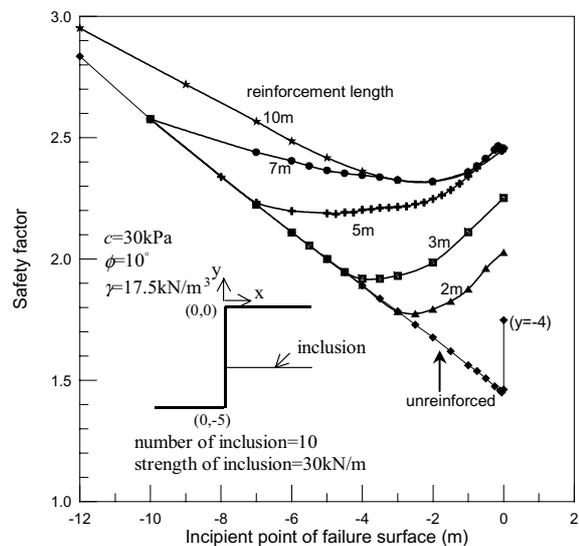


Figure 13. Inclusion length vs. safety factor