

# The research about designing method of vertical reinforced slopes

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**ABSTRACT:** Deformation effect and strength effect of flexible reinforcements and soil-nails are considered, combined with proposed safety factor, the paper gives out a designing methods of vertical reinforced slopes under theory of soil plasticity and limit analysis. The designing method in the paper can also be used in slant reinforced slopes.

## 1 UPPER BOUND HEIGHT $H_u$ OF VERTICAL COHESIVE SLOPE

Chen [1] had given out the upper bound height  $H_u$  of vertical cohesive slope by formula (1) under translational failure mechanism.

$$H_u = \frac{4c}{\rho} \operatorname{tg}(\pi/4 + \psi/2) \quad (1)$$

Where  $c$  and  $\psi$  = given cohesion and friction angle, respectively;  $\rho$  = the soil's unit weight.

To account for reinforcement's contribution, it is assumed that: (1) reinforcements are placed at same space along the slope height; (2) reinforcements have enough anchorage length and can not be pulled out from anchorage soil mass when translational failure happens; (3) flexible reinforcements have zero bend stiffness and can carry only tension force; and (4) soil-nail has certain value of bend stiffness, it can carry tension force and shear force and moment at the same time.

## 2 VERTICAL SLOPE REINFORCED BY FLEXIBLE REINFORCEMENTS

### 2.1 Deformation effect of flexible reinforcements

In fact, slip surface will be a shear zone and has certain thickness, because the flexible reinforcements can only carry tension force, in order to prevent translational failure most effectively, reinforcements at the slip surface will slowly change its orientation so that at the last the orientation of tension force will be opposite to slip velocity  $V$  of soil mass, as shown in Fig.1.

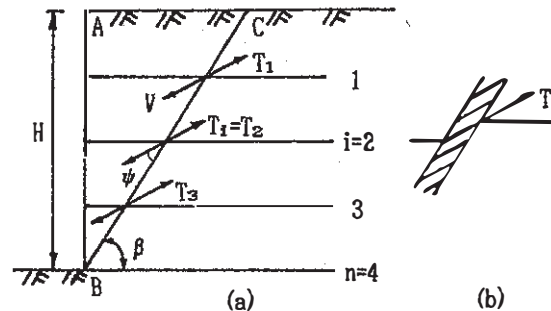


Fig.1. Deformation effect of flexible reinforcements under translational failure

### 2.2 Strength effect of flexible reinforcements

Based on soil plasticity and limit analysis theory, strength effect of reinforcements can be calculated by use of energy work rate form. As shown in Fig.1; strength effect of reinforcements includes two parts: the first is frictional dissipated energy work rate produced between the reinforcements and its surrounding soil, its value is equal to  $\sum_{i=1}^n T_i V \cos(\beta - \psi)$ ,  $n$  is number of reinforcements along the slope height; the second is tensile dissipated energy work rate produced by reinforcements at the slip surface to prevent soil mass moving down, its value is equal to  $\sum_{i=1}^n T_i V$ .

### 2.3 Determination about upper bound horizontal space $S_{hmin}$ of flexible reinforcements

Horizontal space  $S_h$  of reinforcements is taken as calculating length of vertical slope, for slip wedge ABC shown in Fig.1, energy work rate done by soil weight is expressed by formula (2).

$$W_{ex} = 0.5 \rho S_n H^2 V \text{ctg} \beta \sin(\beta - \psi) \quad (2)$$

If soil satisfies Mohr-coulumb failure criteria and obeys associated flowing rule, then interior dissipated work rate  $W_{in}$  is produced by cohesion  $C$  along slip surface BC and by reinforcements, it can be expressed as follows.

$$W_{in} = V [1 + \cos(\beta - \psi)] \sum_{i=1}^n T_i + \frac{CHS_n}{\sin \beta} V \cos \psi \quad (3)$$

The flexible reinforcements are placed at the same space along the slope height, so vertical distance from the  $i$ th-reinforcement to slope top surface is  $(i/n)H$ , and self-weight pressure on the  $i$ th reinforcement is  $(i/n)\rho H$ , corresponding horizontal length of the reinforcement in the slip soil mass is  $(n-i)/n H \text{ctg} \beta$ . Supposed the width of single reinforcement is  $\Delta b$ , the cohesion and friction angle between the reinforcements and its surrounding soil are  $C_b = \alpha_1 C$  and  $\psi_b = \alpha_2 \psi$ , where  $\alpha_1$  and  $\alpha_2$  are some coefficient, and there is no surcharge on the slope top surface, then frictional force or tension force of the  $i$ th reinforcement at the slip surface is expressed as follows.

$$T_i = 2 \frac{i(n-i)}{n^2} \Delta b \rho H^2 \text{tg} \psi_b \text{ctg} \beta + 2 \frac{i}{n} \Delta b H C_b \text{ctg} \beta \quad (4)$$

Putting formula (4) into formula (3) and making  $W_{ex}$  equal  $W_{in}$ , then formula (5) can be obtained.

$$S_n = \frac{4 \Delta b [1 + \cos(\beta - \psi)] \left[ C_b H \sum_{i=1}^n \frac{i(n-i)}{n} \rho H^2 \text{tg} \psi_b + \sum_{i=1}^n \frac{i(n-i)}{n^2} \right]}{\rho H^2 \sin(\beta - \psi) - 2 \alpha_1 C_b \cos \psi \sec \beta} \quad (5)$$

If given  $H$ , then  $S_n$  is a function of  $\beta$ , the range of  $\beta$  chosen is bigger than  $(\pi/4 + \psi/2)$ , formula (5) can be calculated by trial and error, angle  $\beta$  corresponding to  $S_{n \min}$  is called critical failure angle  $\beta_{cr}$  of vertical reinforced slope, and  $S_{n \min}$  is called upper bound horizontal space of reinforcements.

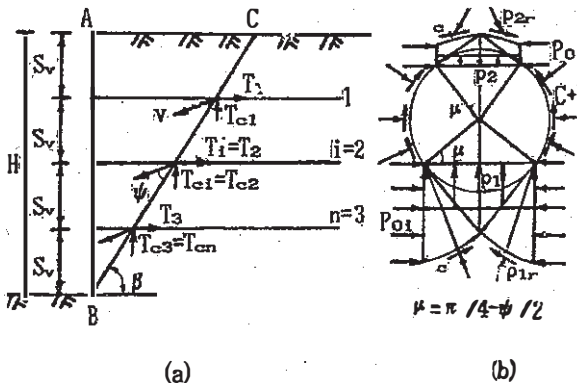


Fig.2. Strength effect of soil-nails under translational failure and flowing resistance mode of soil-nail

### 3 VERTICAL SLOPES REINFORCED BY SOIL-NAILS

#### 3.1 Flowing resistance produced by soil-nails

Space of soil-nails is bigger and its diameter is smaller, when translational failure takes place, the soil mass near the planar slip surface BC often flows around the soil-nails. In order to calculate the flowing resistance produced by soil-nails, mode is established as shown in Fig.2, flowing resistance of per unit soil-nail's length is expressed in formulas (7) and (6).

$$\sigma_{b1} = (P_{o1} + C \text{ctg} \psi) D_s \left[ \frac{\cos(\pi/4 - \psi/2)}{1 - \sin \psi} e^{-(\alpha_2/2 + \psi) \text{ctg} \psi} - \frac{\sin(\pi/4 - \psi/2)}{1 + \sin \psi} e^{-(\alpha_2/2 + \psi) \text{ctg} \psi} + \text{tg} \psi [\cos(\pi/4 - \psi/2) + \sin(\pi/4 - \psi/2)] + [\sin(\pi/4 - \psi/2) - \cos(\pi/4 - \psi/2)] \right] \quad (6)$$

$$P_{o1} = \frac{1}{3} (1 + 2K_0) \rho h_1 \quad (7)$$

Where  $D_s$  = the diameter of grouted soil-nails;  $K_0$  = static horizontal earth pressure coefficient.

#### 3.2 Transverse shear force of soil-nails

Soil flowing around soil-nails is taken place in a small region near the slip surface BC, as aparted from the small region, elastic contact is kepted between soil-nails and its surrounding soil. In order to calculate conservatively and approximately, soil-nail is taken as infinite long beam in the soil mass, by using of boundary conditions of the connected point between the  $i$ th nail and BC, which moment is equal to zero and  $\sigma_{b1}$  is given by formula(6), then transverse shear force  $T_{ci}$  is derived by formula(8) and (9).

$$T_{ci} = L_0 \sigma_{b1} / 2 \quad (8)$$

$$L_0 = 4 \sqrt{4EI / (K_s D_s)} \quad (9)$$

Where  $K_s$  = modulus of subgrade reaction for the soil;  $E$  = Yang's modulus of grouted soil-nail;  $I$  = moment of area of grouted soil-nail.

Calculation results show that angle deformation of soil-nails at slip surface BC can be neglected. So horizontal soil-nails can be considered when translation failure happens.

#### 3.3 Determination about upper bound horizontal space $S_{n \min}$ of soil-nails

Horizontal space  $S_n$  of soil-nails is taken as calculating length of vertical slope, similar to vertical slope reinforced by flexible reinforcements, work rate done by soil weight  $W_{ex}$  also is expressed

by formula(2), the tension force or friction force  $T_i$  of the  $i$ th soil-nail is expressed by formula(10).

$$T_i = \pi D_a \frac{i[(n+1)-i]}{(n+1)^2} \rho H^2 \text{ctg} \beta \text{tg} \psi + \pi D_a \frac{(n+1)-i}{n+1} H \text{ctg} \beta C_s \quad (10)$$

Corresponding dissipated work rate produced by interior forces is expressed by formula(11).

$$W_{in} = 2V \cos(\beta - \psi) \sum_{i=1}^n T_i + V \sin(\beta - \psi) \sum_{i=1}^n T_{ci} + \frac{CHS_n}{\sin \beta} V \cos \psi \quad (11)$$

Making  $W_{ex}$  equal to  $W_{in}$ , formula (12) can be obtained.

$$S_n = (4\pi D_a \cos(\beta - \psi) [C_s H \sum_{i=1}^n \frac{n+1-i}{n+1} + \rho H^2 \text{tg} \psi \text{tg} \beta] + \sum_{i=1}^n \frac{i(n+1-i)}{(n+1)^2} + 2 \sin(\beta - \psi) \text{tg} \beta \sum_{i=1}^n T_{ci}) / [\rho H^2 \sin(\beta - \psi) - 2CH \cos \psi \sec \beta] \quad (12)$$

Formula (12) can be calculate by trial and error, angle  $\beta$  corresponding to the  $S_{hmin}$  is called critical failure angle  $\beta_{cr}$  of vertical slope reinforced by soil-nails, and  $S_{hmin}$  is called upper bound horizontal space of soil-nails.

#### 4 DESIGNING METHOD OF VERTICAL REINFORCED SLOPES

##### 4.1 Determination about value of safety factor

The  $S_{hmin}$  can be derived by formulas(5) or (12) by use of upper bound solution method, in fact, the given slope height  $H$  in formulas (5) or (12) also is the upper bound  $H_u$  of slope. Obviously, we can not directly use  $n$  and  $H$  and  $S_{hmin}$  to design reinforced slopes, so how to design reinforced slopes under working load still need to discuss, this is related to safety factor chosen and its value used. Safety factor is defined as  $\tau = c/k + \sigma$  ( $\text{tg} \psi / k$ ), which cohesion  $C$  and friction angle  $\psi$  of soil will be dropped to  $c_s = c/k$  and  $\psi_s = \arctg(\text{tg} \psi / k)$ , respectively. Next, we discuss the value of safety factor.

Chen[1] gives low bound  $H_{cr}$  of vertical cohesive slope, as expressed by formula(13).

$$H_{cr} = \frac{2c}{\rho} \text{tg}(\pi / 4 + \psi / 2) \quad (13)$$

The  $H_{cr}$  is similar to  $P_{cr}$  of shallow foundation engineering,  $P_{cr}$  also is the low bound solution of limit bearing pressure, So  $P_{cr}$  can be taken as designing pressure conservatively under working load. Similar to  $P_{cr}$ , the  $H_{cr}$  can also be taken as designing height of vertical cohesive slope.

If low bound  $H_{cr}$  is given out by upper bound  $H_u$ ,  $c$  and  $\psi$  in the formula (1) must be reduced, corresponding safety factor is defined as standard safety factor  $K_k$ , then formula (14) is set up.

$$\frac{4c}{\rho K_k} \text{tg}[\pi / 4 + 0.5 \arctg(\text{tg} \psi / K_k)] = \frac{2c}{\rho} \text{tg}(\pi / 4 + \psi / 2) \quad (14)$$

Rearranging formula (14), then formula (15) will be obtained.

$$\frac{2}{K_k} \text{tg}[\pi / 4 + 0.5 \arctg(\text{tg} \psi / K_k)] = \text{tg}(\pi / 4 + \psi / 2) \quad (15)$$

$K_k$  is the function of  $\psi$ . when  $\psi = 0, K_k = 2$ ; when  $\psi \neq 0, K_k < 2$ , and  $K_k$  is decreased as  $\psi$  increased.

The  $K_k$  can be taken as initial value of reinforced slope, because reinforcements make  $C$  and  $\psi$  increased, by formula (15) we know that the  $k$  of vertical reinforce slope is smaller than  $K_k$  of cohesive vertical slope, it is shown that difference between upper bound  $H_u$  and low bound  $H_{cr}$  of vertical reinforced slope will be reduced, so by use of coefficient  $\alpha_3$ , safety factor  $k$  of vertical reinforced slope is expressed in formula (16).

$$K = \alpha_3 K_k \quad (16)$$

$\alpha_3$  can be affected by many factors and is difficult to be determined, in order to be briefly, we take  $K = K_k$ .

##### 4.2 Designing procedure of vertical reinforced slope

Reinforcements are placed in horizontal and have enough anchorage length, in order to prevent reinforcements pulled out from anchorage soil mass, anchored length of reinforcements should be equal to or bigger than reinforcement's length in slip soil mass, so  $i$ th flexible reinforcement's length can be expressed by formula (17).

$$L_i > 2 \frac{n-i}{n} H \text{ctg} \beta_{cr} \quad (17)$$

$\beta_{cr}$  can be gotten by putting  $C_s$  and  $\psi_s$  into formula (5).

The  $i$ th soil-nail's length can be expressed by formula (18).

$$L_i > 2 \frac{n+1-i}{n+1} H \text{ctg} \beta_{cr} \quad (18)$$

$\beta_{cr}$  of formula (18) can also be obtained by putting  $C_s$  and  $\psi_s$  into formula (12).

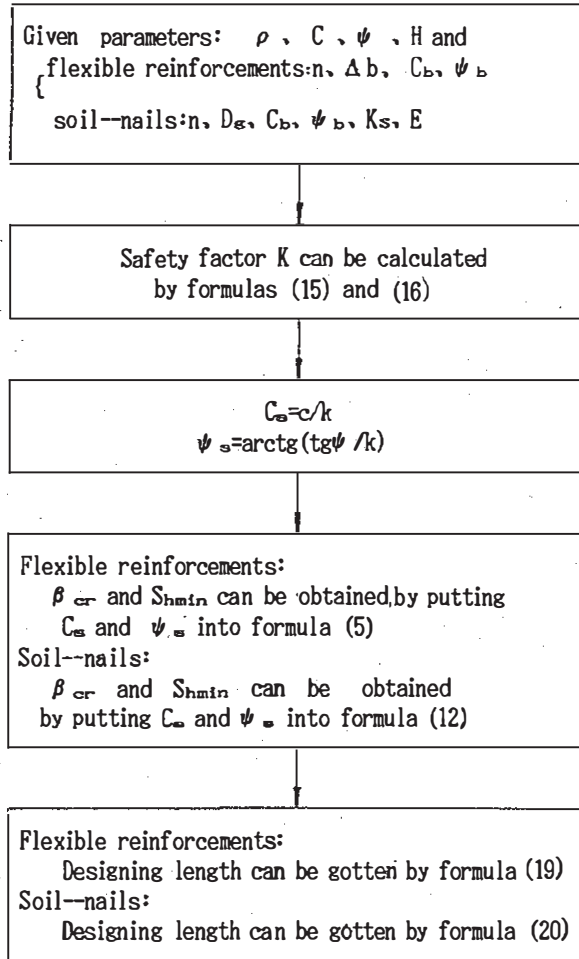
Because the  $L_1$  is maximum, in order to construct conveniently,  $L_1$  is defined as general reinforcement's length. For flexible reinforcements,  $L_1$  is expressed by formula (19)

$$L_1 = 2 \frac{n-1}{n} H \text{ctg} \beta_{cr} \quad (19)$$

For soil-nails,  $L_1$  is expressed by formula(20).

$$L_1 = 2 \frac{n}{n+1} H \text{ctg} \beta_{cr} \quad (20)$$

Designing procedure diagram of vertical reinforced slope is shown as follows.



## 5 CALCULATION EXAMPLE

### 5.1 Calculation example about vertical slope reinforced by flexible reinforcements

Some vertical slope,  $\rho = 18 \text{KN/m}^3$ ,  $c = 15 \text{kPa}$ ,  $\psi = 25^\circ$ ,  $H = 4.10 \text{m}$ ,  $\Delta b = 0.03 \text{m}$ ,  $n = 6$ . In order to consider different effects, two plans are adopted: the first is  $C_b = C$  and  $\psi_b = \psi$ ; the second is  $C_b = 0$  and  $\psi_b = \psi$ . Calculation results of the two plans are listed in table 1.

Based on the results of table 1, the following conclusions can be obtained.

- ① Reinforcements increase critical slip angle  $\beta_{cr}$  and make slope shallow translational failure.
- ② Cohesion  $C_b$  between reinforcements and its surrounding soil has bigger effect on  $S_{hmin}$ .
- ③ The  $n$ th flexible reinforcement also is important, it increases the low bound  $H_{cr}$  and stability of

vertical reinforced slopes.

Table 1 Calculation results

Result Plans	$k$	$\beta_{cr}$	Designing $S_{hmin}$	Designing $L_1$
$C_b = c$ $\psi_b = \psi$	1.68	$60^\circ$	0.50m	3.94m
$C_b = 0$ $\psi_b = \psi$	1.68	$60^\circ$	0.24m	3.94m

### 5.2 Calculation example about vertical slope reinforced by soil-nails

Some vertical slope,  $\rho = 16.3 \text{KN/m}^3$ ,  $c = 19 \text{kPa}$ ,  $\psi = 36.5^\circ$ ,  $K_s = 50000 \text{KN/m}^3$ ,  $H = 8.0 \text{m}$ ,  $n = 3$ , diameter  $D$  of steel bar  $= 0.025 \text{m}$ ,  $D_s = 0.10 \text{m}$ , modulus  $E$  of steel  $= 210 \times 10^9 \text{KN/m}^2$ , modulus  $E$  of concrete  $= 2.6 \times 10^7 \text{KN/m}^2$ . In order to consider different effects, two plans are adopted: the first is  $T_{ci} \neq 0$ ; the second is  $T_{ci} = 0$ . Calculation results are listed in table 2.

Calculation results show that the designing horizontal space  $S_{hmin}$  of soil-nails will be increased if transverse shear force  $T_{ci}$  and bend stiffness are considered,  $S_{hmin}$  is conservative when  $T_{ci}$  is equal to zero, conservative degree is  $(2.02 - 1.76) / 1.76 = 14.8\%$ , thus the plan which  $T_{ci}$  is equal to zero is approximate reasonable designing method for vertical slope reinforced by soil-nails.

Table 2 Calculation results

Result Plans	$K$	$\beta_{cr}$	Designing $S_{hmin}$	Designing $L_1$
$T_{ci} \neq 0$	1.58	$60^\circ$	2.02m	6.93m
$T_{ci} = 0$	1.58	$60^\circ$	1.76m	6.93m

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