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Theoretical earth pressure distribution on retaining wall with reinforced earth backfill

Distribution théorique des pressions derrière un mur soutenant un massif armé

Les auteurs proposent une méthode de calcul de la poussée s'exerçant sur un mur retenant un remblai armé par des bandes horizontales. On suppose que le mouvement du mur est suffisant pour que se développe dans le remblai un coin de Coulomb et l'on fait en outre l'hypothèse que le frottement sol-armatures est complètement mobilisé. On obtient une réduction appréciable de la poussée qui est fonction des propriétés du sol et du nombre des armatures. La longueur optimale des armatures est fonction de la hauteur et l'on donne la répartition des pressions sur le mur ainsi que le point de passage de la résultante.

INTRODUCTION

Reinforced earth, since its invention by Henri Vidal in the Sixties have found largest application in the construction of retaining walls. These consist of a skin of pre-cast concrete panels or semi-elliptical metallic units to which strips of suitable material (usually galvanized steel, aluminium alloy or heavy duty synthetic fabrics) are connected at suitable horizontal and vertical spacings. The stresses in the soil are transferred to the reinforcing strips through friction developing between the soil and strips, and the anchorage provided by the strips imparts the stability to the retaining wall.

The construction of such reinforced earth retaining walls, especially those employing pre-cast concrete panels for skin, requires equipment for handling of heavy pre-cast panels. Such equipment is not readily available to the construction agencies in developing countries. Further, the artisans in such countries, especially those in semi-urban and rural areas, may not be able to learn the technique of construction of these walls because of their back ground. There is, therefore, a need for modifying the technique of construction of reinforced earth retaining walls to suit the resources and competence of technical man power of developing countries.

The drawback listed above can be overcome by providing a reinforced earth backfill behind a conventional retaining wall of masonry or R.C.C. The reduction in active pressure due to this type of backfill will lead to thinner section of retaining walls and overall economy in construction.

ANALYSIS

A retaining wall of height H, with a vertical back, retains a horizontal backfill. The cohesionless backfill soil has friction angle ϕ and dry unit weight γ. It is reinforced with horizontally placed strips of length L and width w, starting from the back face, at vertical and horizontal spacings of ΔH and b respectively. It is assumed that the wall moves laterally under the influence of lateral thrust and that a Coulomb wedge of reinforced soil separates from the rest of the reinforced earth backfill when active conditions are reached. Full frictional resistance is assumed to be mobilised along the reinforcing strips at failure, the total frictional resistance offered by a strip being computed from its effective length, which is lesser of the two portions DE and EF - one inside and the other outside the failure wedge ABC. The failure surface BC makes an angle θ with the vertical.

Considering the equilibrium of an element P'Q'R'S' in the wedge of thickness dy and located at a distance y from top, the following forces can be identified* per unit length of wall (Saran and Prakash, 1970) Fig. 1.

p_y : Pressure intensity acting uniformly on P'Q' in the vertical direction.

$p_y + dp_y$: Intensity of uniform reaction acting on R'S' in the vertical direction

p_θ : Reaction intensity on Q'R', acting at an angle ϕ to the normal to Q'R'

p : Pressure intensity on P'S' acting at an angle δ with the normal to P'S'

W : Weight of slice P'Q'R'S' acting downwards

In addition there will be tensile forces T in the strips passing through the element P'Q'R'S'.

(1) Balancing all the forces acting on the slice in the vertical direction :

$$p_y(H-y)\tan\theta - (p_y+dp_y)(H-y-dy)\tan\theta + 0.5\gamma dy.(H-y+H-y-dy)\tan\theta - p.dy\sin\delta - p_\theta\, dy\sec\theta.\sin(\theta+\phi) = 0.$$

Neglecting small quantities of second order, the equation reduces to

$$\frac{dp_y}{dy} = \frac{p_y+(H-y)\gamma}{(H-y)} - \frac{p\sin\delta}{(H-y)\tan\theta} - \frac{p_\theta \sec\theta\sin(\theta+\phi)}{(H-y)\tan\theta} \quad \ldots\ldots (1)$$

(2) Balancing all the forces acting on the slice in the horizontal direction :

$$\frac{T}{\Delta H}.dy + p dy.\cos\delta - p_\theta dy\ \sec\theta.\cos(\theta+\phi) = 0,$$

assuming that the tension in the strips is uniformly distributed over the vertical distance ΔH. This equation simplifies to

$$p_\theta = \frac{t+p\cos\delta}{\sec\theta.\cos(\theta+\phi)} \quad \ldots\ldots (2)$$

where $t = T/\Delta H$

(3) Taking moments of all the forces about the mid point of slice between Q' and R':

$$p_y(H-y)\tan\theta\left[\frac{(H-y)}{2}\tan\theta - \frac{dy}{2}\tan\theta\right] - (p_y+dp_y).\left[(H-y-dy)\tan\theta\right]\left[\frac{(H-y-dy)}{2}.\tan\theta + \frac{dy}{2}\tan\theta\right] - p\ dy\ \sin\delta\left[(H-y-\frac{dy}{2})\tan\theta\right] + 0.5\gamma dy(H-y-\frac{dy}{2})^2.\tan^2\theta = 0$$

Simplifying and neglecting small quantities of higher order

$$\frac{dp_y}{dy} = \gamma - \frac{2p\sin\delta}{(H-y)\tan\theta} \quad \ldots\ldots (3)$$

Substituting the value of p_θ in Eq.(2), it reduces to

$$\frac{dp_y}{dy} = \frac{p_y+(H-y)\gamma}{(H-y)} - \frac{p\sin\delta}{(H-y)\tan\theta} - \frac{(t+p\cos\delta)\tan(\theta+\phi)}{(H-y).\tan\theta} \quad \ldots\ldots (4)$$

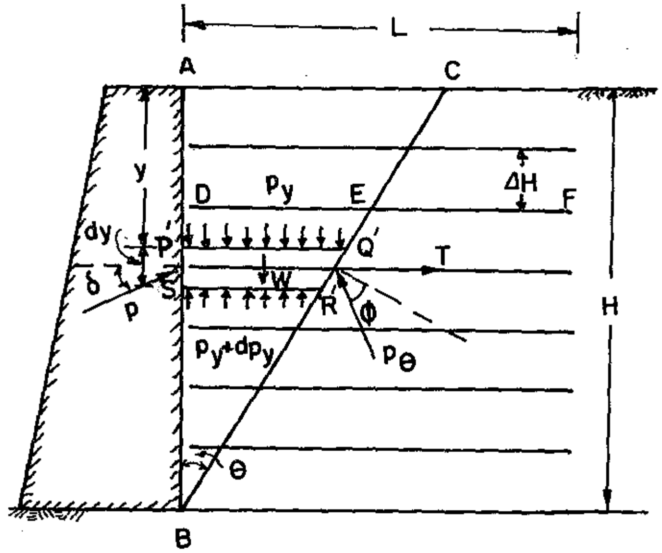

Fig. 1. Forces acting on the element P'Q'R'S'

Equating Eq.(3) and Eq.(4) results in

$$p = p_y.\frac{\tan\theta}{\cos\delta\tan(\theta+\phi)-\sin\delta} - \frac{t.\tan(\theta+\phi)}{\cos\delta.\tan(\theta+\phi)-\sin\delta}$$

$$\text{or } \frac{dp}{dy} = \frac{dp_y}{dy}.\frac{\tan\theta}{\cos\delta\tan(\theta+\phi)-\sin\delta} - \frac{dt}{dy}.\frac{\tan(\theta+\phi)}{\cos\delta\tan(\theta+\phi)-\sin\delta}$$

Substituting for dp_y/dy from eq.(3) and simplifying

$$\frac{dp}{dy} = C_2\gamma - C_1.\frac{p}{H-y} - C_3\frac{dt}{dy} \quad \ldots\ldots (5)$$

where $C_1 = 2\sin\delta.\cos(\theta+\phi)$

$$C_2 = \frac{\sin\theta\cos(\theta+\phi)}{\cos\theta.\sin(\theta+\phi-\delta)}$$

$$C_3 = \frac{\sin(\theta+\phi)}{\sin(\theta+\phi-\delta)}$$

The tension in the strips T can be assumed as

$$T = 2.w.f.\sigma_v.l'/b$$

where w = width of strip

l' = effective length of strip

f = soil-strip coefficient of friction

σ_v = vertical stress

b = horizontal spacing of strips

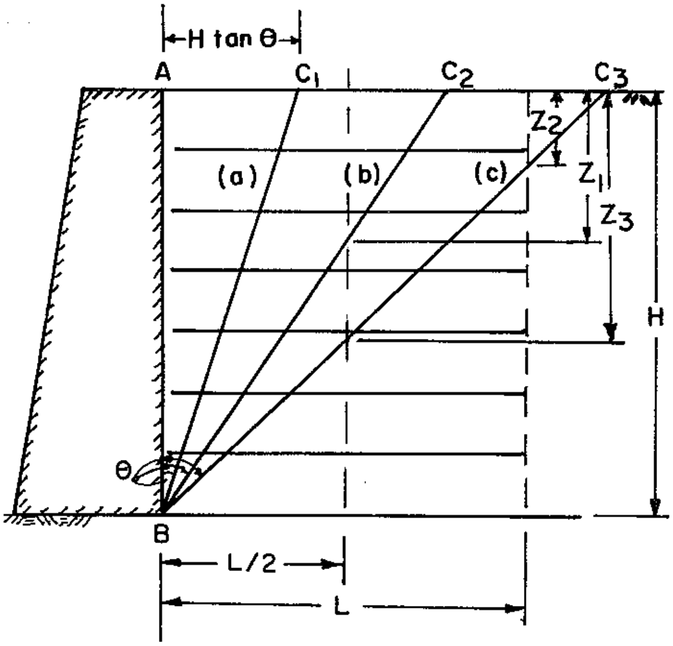

Fig.2 - Variation of effective length of strips with angle θ

Assuming $\sigma_v = (y + \frac{dy}{2})\gamma$

$T = 2w.f.\gamma(y + \frac{dy}{2})l'/b$

The value of l' will vary from strip to strip and will depend upon angle θ and length L. Three cases may arise (Fig.2)

Case (a) : $H \tan\theta \leq \frac{L}{2}$. In this case effective length $l' = (H-y)\tan\theta$ for all strips.

Case(b): $L/2 < H \tan\theta \leq L$. In this case for depth $y = z_1$, $l' = L-(H-y)\tan\theta$, and for depth $y > z_1$ $l' = (H-y)\tan\theta$

Case (c): $L < H \tan\theta$

Here $l' = 0$ for $y \leq z_2$,

$l' = L-(H-y)\tan\theta$ for $z_2 < y < z_3$ and

$l' = (H-y)\tan\theta$ for $y > z_3$

The expressions for p, total pressure P and centre of pressure for the three cases are determined as below :

Case (a): $T = 2wf\gamma(y + \frac{dy}{2})(H-y)\tan\theta/b$

or $t = 2wf\gamma(y+0.5.dy)(H-y)\tan\theta/b.\Delta H$

$\therefore \frac{dt}{dy} = k(H-2y)$ where $k = \frac{2wf\gamma\tan\theta}{b.\Delta H}$, neglecting small quantities of second order.

Equation (5) for case (a) is

$$\frac{dp}{dy} = -C_1\frac{p}{H-y} + C_2\gamma - C_4(H-2y) \quad .... (6)$$

Where $C_4 = C_3.k$

The solution of the differential equation for the boundary condition $p = 0$ at $y = 0$ is

$$\bar{p} = \frac{p}{\gamma H} = \frac{-C_2}{1-C_1}\left[(1-y/H)-(1-y/H)^{C_1}\right] + \frac{2C_4H}{\gamma(2-C_1)}\left[(1-y/H)^2-(1-y/H)^{C_1}\right] + \frac{C_4H}{\gamma(1-C_1)}\left[(1-y/H)^{C_1}-(1-y/H)\right] \quad .... (7)$$

The total pressure $P = \int_0^H p dy = 0.5\,\gamma H^2.$

$$\left[\frac{1.0}{(C_1+1.0)} . (C_2 - \frac{C_4H}{3\gamma})\right] \quad .... (8)$$

The distance of action of P from the base, $\bar{h}$, is

$$\bar{h} = 1.0 - \frac{\int_0^H py\,dy}{\int_0^H p\,dy} = \frac{(2\gamma HC_2 - C_4H^2)(C_1+1.0)}{(C_1+2)(3\gamma C_2 - C_4H)} \quad .... (9)$$

Case (b) : For $y \geq z_1$, the expression for $\bar{p}$ will be the same as obtained in case (a). For $y < z_1$, the effective length of strip to be considered will be $[L-(H-y)\tan\theta]$. The corresponding expression for t, then, is

$$t = k(y + \frac{dy}{2})\left[\frac{L}{\tan\theta} - (H-y-\frac{dy}{2})\right] \text{ and}$$

$$\frac{dt}{dy} = k(\frac{L}{\tan\theta} - H - 2y)$$ ignoring derivatives second order.

Equation (5) gets modified to

$$\frac{dp}{dy} = -C_1\frac{p}{H-y} + C_2\gamma - C_4(\frac{L}{\tan\theta} - H + 2y) \quad ...(10)$$

where $C_4 = k.C_3$ as before.

The solution of equation (10) is

$$\bar{p} = \frac{p}{\gamma H} = \left[\frac{C_4\;.L/\tan\theta}{\gamma(1.0+C_1)} - \frac{C_2}{1.0-C_1} + \frac{C_4H}{\gamma(1.0-C_1)}\right].\left[(1-\frac{y}{H})-(1-\frac{y}{H})^{C_1}\right] + \frac{2C_4H}{\gamma(2-C_1)}\left[(1-\frac{y}{H})^{C_1}-(1-\frac{y}{H})^2\right] \quad .... (11)$$

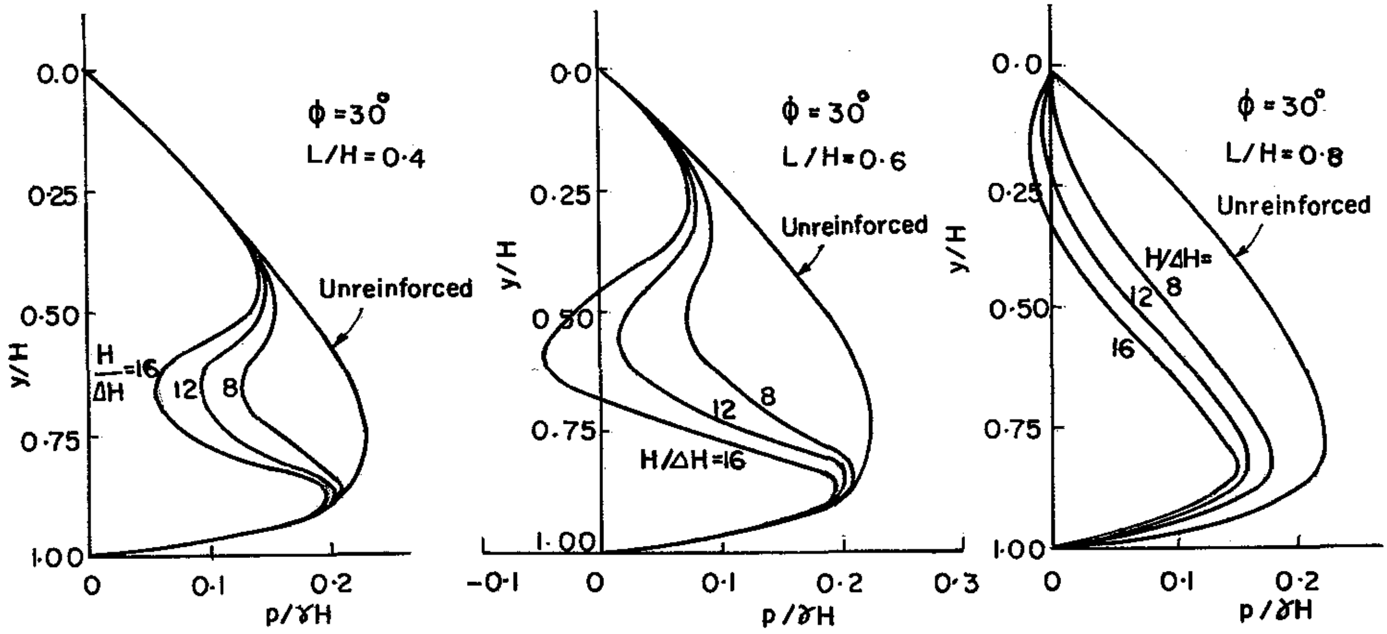

Fig. 3 Pressure Distribution on the Retaining Wall for different lengths and spacings of strips.

For computing the total pressure P, the appropriate expressions of p (Equations 11 and 7) are integrated over the corresponding domains. The final expression is

$$P = \frac{1}{C_1+1} \cdot Q^{C_1+1}\left[K_1+K_2-2K_3\right] + \frac{1}{3}\,Q^3\left[2K_3\right] - \frac{1}{2}\,Q^2(K_1+K_2) + \frac{1}{C_1+1}(K_3-K_1) + \left(\frac{K_1}{2} - \frac{K_3}{3}\right) \quad \ldots(12)$$

$$K_1 = \frac{\gamma H^2}{(1-C_1)}\left[\frac{C_4 \cdot L}{\gamma \tan\theta} - C_2 + \frac{C_4 H}{\gamma}\right]$$

$$K_2 = \frac{\gamma H^2}{1-C_1}\left[C_2 + \frac{C_4 H}{\gamma}\right]$$

$$K_3 = \frac{2\,C_4 H^3}{2-C_1} \quad \text{and} \quad Q = \frac{L \cot\theta}{2H}$$

The point of action of resultant earth pressure above the base $\bar{h}$ is

$$\bar{h} = \left[(M_1+M_2+2M_3)\frac{Q^{C_1+2}}{C_1+2} + 2M_3\frac{Q^4}{4} - (M_1+M_2)\frac{Q^3}{3} - (M_1-M_3)\frac{1}{C_1+2} + \left(\frac{M_1}{3} - \frac{M_3}{4}\right)\right] \Big/ \left[\frac{1}{H}\left\{(M_1+M_2-2M_3)\cdot\frac{Q^{C_1+1}}{C_1+1} + 2M_3\frac{Q^3}{3} - (M_1+M_2)\frac{Q^2}{2} + (M_3-M_1)\frac{1}{C_1+1} + \frac{M_1}{2} - \frac{M_3}{3}\right\}\right] \quad \ldots\ (13)$$

Where

$$M_1 = \frac{C_4 L_1 H^3}{(1-C_1)\tan\theta} - \frac{C_2 H^3 L}{1-C_1} + \frac{C_4 H^4}{1-C_1}$$

$$M_2 = \frac{1}{1-C_1}\left[C_2 \gamma H^3 + C_4 H^4\right], \quad M_3 = \frac{2C_4 H^4}{2-C_1}$$

Case (c) For the condition $H\tan\theta \geq L$, the expression for P will be obtained by integrating the appropriate expressions for p over three different depths, namely y=0 to $y = z_2$, $y = z_2$ to $y = z_3$ and $y = z_3$ to y = H. The expression for p for the domain y=0 to $y=z_2$ is

$$p = \gamma H \frac{C_2}{1-C_1}\left[\left(1-\frac{y}{H}\right)^{C_1} - \left(1-\frac{y}{H}\right)\right] \quad \ldots,\ (14)$$

as $C_4 = 0$ in this case.

For the other two domains, equation 11 and 7 will be applicable. The final value of P is

$$P = \frac{Q^{C_1+1}}{C_1+1}\left[K_1(1-2^{C_1+1}) + K_2 + K_3(2^{C_1+1}-2) - K_4 \cdot 2^{C_1+1}\right] - Q^3\left[2K_3\right] + Q^2\left[\frac{3}{2}K_1 - \frac{1}{2}K_2 + 2K_4\right] + K_4\left(\frac{1}{C_1+1} - \frac{1}{2}\right) \quad \ldots,\ (15)$$

where $K_4 = \gamma C_2 H^2/(1-C_1)$

The expression for $\bar{h}$ for this case has also been derived.

PARAMETRIC STUDY

A parametric study has been performed for a 4m high retaining wall supporting a backfill reinforced with steel strips 6 cm wide, spaced 0.5m apart horizontally. Coefficient of soil-strip friction is assumed as 0.4. The total earth pressure is maximised by varying angle θ. The incipient failure is presumed as due to lack of adherence between soil and reinforcement. The parameters studied are listed in Table 1.

Table 1 Parameters Studied .

Parameter	Range	Interval
Ø	20° - 40°	5°
L/H	0.0,0.3-1.0	0.1
H/ΔH	8 - 16	4

Angle δ was taken equal to 2Ø/3

RESULTS AND DISCUSSIONS

The results of the investigations are presented for a representative value of $Ø = 30°$.

i. Pressure Distribution : Typical plots of pressure distribution for three different cases of reinforced backfill and unreinforced backfill are shown in Fig. 3. It is evident from these that pressure distribution is significantly affected by the reinforcement. Pressure intensity varies with length and number of strips. Negative pressures develop towards the top of the backfill for longer strips and for closer vertical spacing. The pressure distribution pattern for L/H = 0.8 more or less follows the pattern of pressure distribution for unreinforced soil, but for values of L/H = 0.4 and 0.6 , the pattern is different. The pressure falls off considerably in the mid part for these cases while maintaining virtually the same intensity towards the top and bottom of the backfill.

This can be explained by the fact that for the first two cases the failure wedge is of larger size and the strips near the top of backfill do not contribute any friction resistance as their effective length is very nearly zero. In the mid portion the effective length of strip is large and their frictional resistance effectively reduces pressure p. For nearly uniform reduction in pressure along the height of the wall the length of strips in the upper portion should approach the height of wall.

ii. Total Pressure : The total active pressure acting on the wall is represented in non dimensional form by $\bar{P} = P/(0.5\gamma H^2)$ in Fig.4. The total pressures reduce by 28 to 60% for strips of length 0.6 H for different values of ΔH. As is evident, there is an optimum length of strips for each vertical spacing beyond which there is no further decrease in total pressure. For the case under study this optimum length lies between 0.6H to 0.8H.

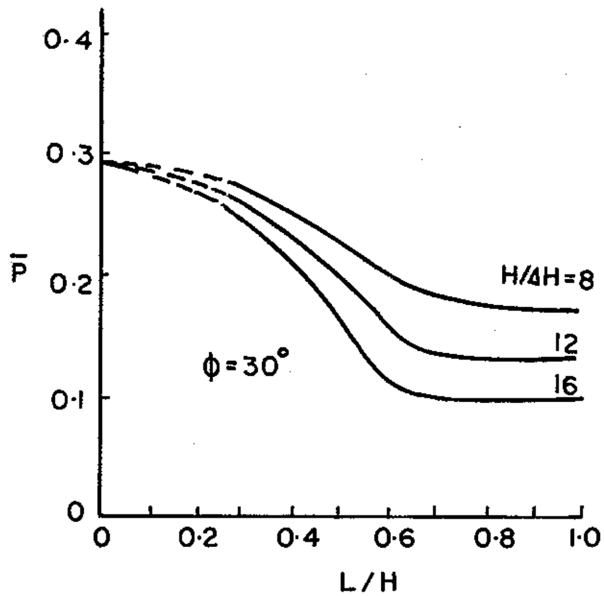

Fig. 4 Non-dimensional Total pressure $\bar{P}$ acting on the retaining wall.

iii. Point of Application of Resultant Pressure : The variation in the height of point of action of resultant earth pressure above the base of retaining wall is plotted in Fig.5, which indicates that after a slight increase in $\bar{h}$ at L = 0.3H it decreases sharply and reaches minimum attainable values corresponding to the optimum length of strips. The decrease in resultant pressure

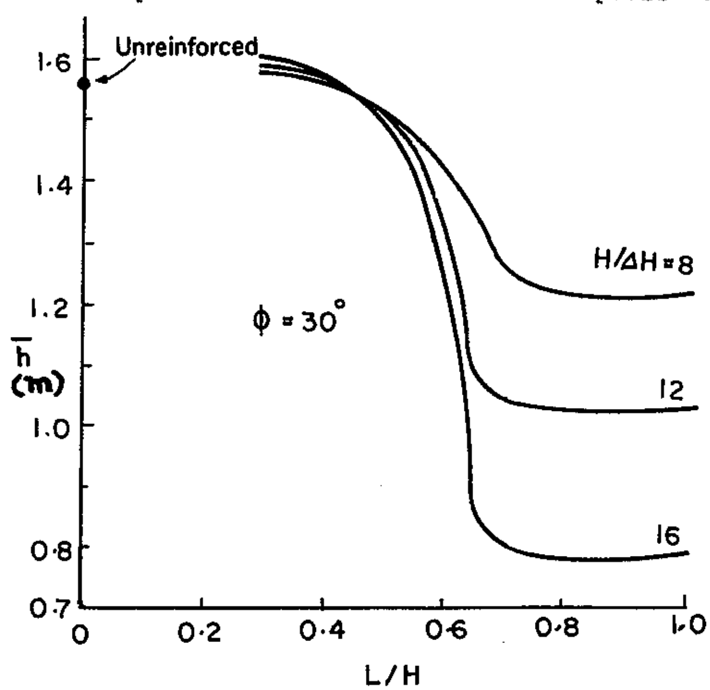

Fig. 5 Variation of height of point of action of resultant pressure on the retaining wall.

P, in conjunction with lower values of $\bar{h}$ should lead to substantial decrease in the moments acting on the base of retaining wall.

iv. Tensile Forces in the Strips : Maximum stresses would develop in the strips at mid height. In case of backfill with $H/\Delta H = 8$ and strip lengths $L = 0.4$ H and 0.6 H, the maximum tensile stress is 0.25t and for $L = 0.8H$ the maximum stress is 0.155t. The computed thicknesses of the 6 cm wide steel strips for these cases work out to 0.3 and 0.19 mm respectively for an allowable stress of 14 kg/mm^2.

Results similar to those described above were obtained for other values of ϕ .

CONCLUSIONS

The pressure acting on the back of a retaining wall and the moments acting on its base can be appreciably reduced by reinforcing the backfill with suitably designed reinforcement. The stresses in the strips are usually small and strips of very small cross-section can be employed. The performance of the strips can be improved by increasing the soil-reinforcement friction by use of grooved or ribbed strips. The above technique of reinforcing backfill can be suitable for developing countries.

REFERENCE

Saran, S. and Prakash, A.(1970),"Siesmic Pressure Distribution in Earth Retaining Walls", Fourth European Symposium on Earthquake Engineering, Sofia, Bulgaria.