Earth Reinforcement, Ochiai, Yasufuku & Omine (eds) © 1996 Balkema, Rotterdam. ISBN 90 5410 833 9

Use of double wedge equilibrium for reinforced earth structures design

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ABSTRACT: Application of the double wedge equilibrium method to Reinforced Earth is analysed together with the way safety coefficients or weighing factors should be taken into account. A comparison of the three most common design methods shows consistent results provided that the safety is correctly implemented and the type of structure is compatible with the domain of applicability of the method.

1. INTRODUCTION

Several methods have been used in the past for designing or for checking the safety of a given REINFORCED EARTH structure (Fig. 1):

1: the most widely used world-wide is certainly the « coherent gravity method » first presented in the « Recommandations and Rules of the Art for Reinforced Earth» (French Ministry of Transport, 1979). In this method the design is done for each layer of strips, layer by layer, using the position of the line of maximum tension which has been found from measurements on full scale structures of a similar shape. The tensile force in the strips at the maximum is computed from the stress distribution in the mechanically stabilised soil mass; section of strips is checked; then adherence capacity behind the line of maximum tension. This method has now been incorporated in many codes both in Europe (AFNOR 1992) and in North America.

2: an other widely used method is the « global stability method » using slip surfaces. It is adapted from slope stability analysis such as the modified slip circle method from Bishop (1955) or the log-spiral (Taylor 1937). The resistance of the strips intersected by the slip surface is taken into account as the minimum of their resistance and their adherence capacity beyond the slip surface. Details of such methods have been presented by Schlosser et al. (1984) and implemented in their program TALREN. The National Project Clouterre (Schlosser et al. 1993) exhibited good agreement between these computations and the actual behaviour of experimental soil-nailing structures.

3: in Germany an other method, close in its approach to the « global stability », has been used for soil nailing. It is both simple and fits well the German habits and code. This global method presented by Gässler (1988) considers the statical equilibrium of two wedges or double wedge.

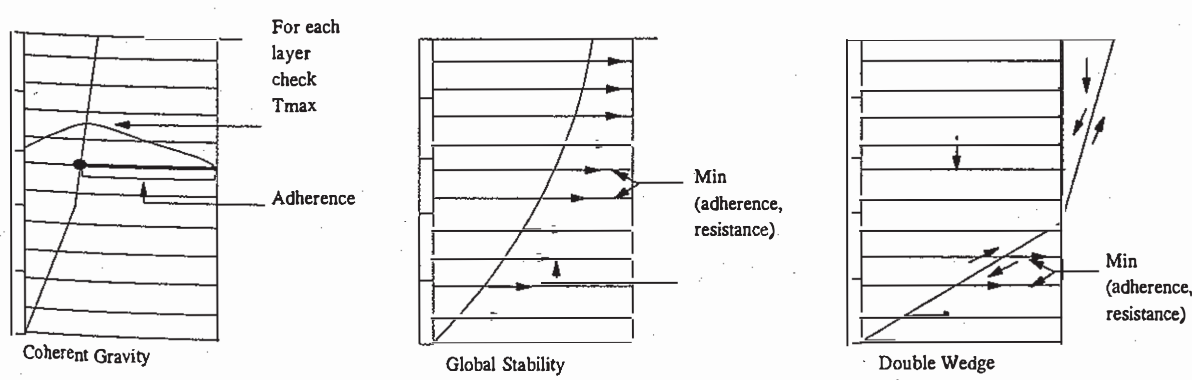

Fig. 1 Various design methods for MSE walls

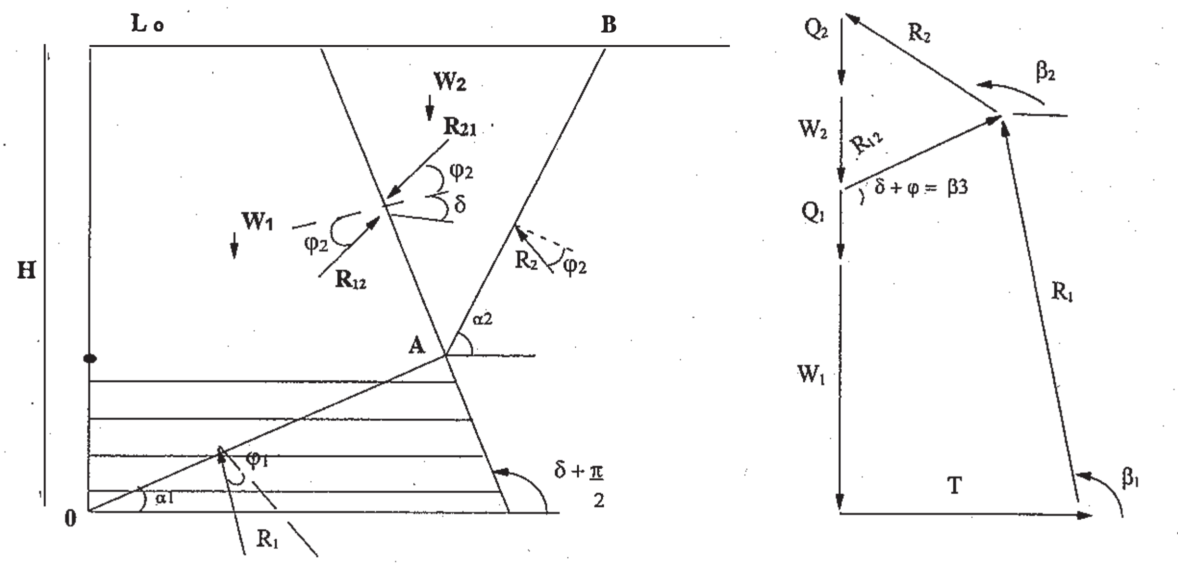

Fig.2 Double wedge static equilibrium

2. EQUILIBRIUM OF THE DOUBLE WEDGE

Figure 2. presents a typical cross section of a structure with an example set of wedges. The limit between the two wedges is assumed to be located at the end of the strips. Hence, it would be vertical in most cases but could, as in figure 2., be inclined. The system is entirely determined once point O, angles α_1 and α_2 are set.

Wedge 1 is in equilibrium under :

- Q1 : the traffic surcharge on top of wedge 1
- W1 : its own weight
- R1 : the interaction between the wedge and the underlying soil.
- R12 : the interaction with wedge 2
- T : sum of the strip tensions ($T = \Sigma(T_i)$)

Wedge 2 is in equilibrium under :

- Q2 : the traffic surcharge on top of wedge 2
- W2 : its own weight
- R2 : the interaction between the wedge and the adjacent soil.
- R21 : the interaction with wedge 1

The friction in the soil results in R1 forming an angle $\phi_1{}^*$ with the normal to OA, R2 an angle $\phi_2{}^*$ with the normal to AB and R21 an angle $\phi_2{}^*$ with the common limit of the two wedges. Let T* be the necessary tension in the strips for equilibrium. The problem can be solved by vertical and horizontal projection of the force diagram (Fig 2, right).

3. FACTOR OF SAFETY :

The computation presented above may be used to check the safety of the structure. Several approaches may be contemplated :

1. Safety on the soil : in this case the tensile force T_i for each strip layer is computed as the minimum of its tensile resistance and the mobilisable friction beyond the wedge. T is the sum of all the Ti. Then for each set of geometry, uniquely defined by $\{O,\alpha_1, \alpha_2\}$, it is possible to compute λ in such a way that the system is just in equilibrium with $(T,\phi_1{}^*,\phi_2{}^*)$ where :

$$\phi_1{}^* = \tan^{-1}(\tan\phi_1/\lambda)$$
$$\phi_2{}^* = \tan^{-1}(\tan\phi_2/\lambda)$$

FS, the factor of safety is defined as the minimum value of λ for all the possible $\{O,\alpha_1, \alpha_2\}$.

2. Safety on the strips : in this case the tensile force T, as computed from the above Ti values, is divided by a coefficient λ to obtain T*. λ is chosen such that the system is just in equilibrium with (T^*,ϕ_1, ϕ_2). FS, the factor of safety is defined as the minimum value of λ for all the possible $\{O,\alpha_1, \alpha_2\}$.

Normally both approaches will not lead to the same value for the factor of safety FS.

This observation will trigger lengthy discussions as to which definition is best and, in the event that the owner of the structure under design has set a target value for FS, may lead to disputes between the client, the contractor and the supplier (of Reinforced Earth).

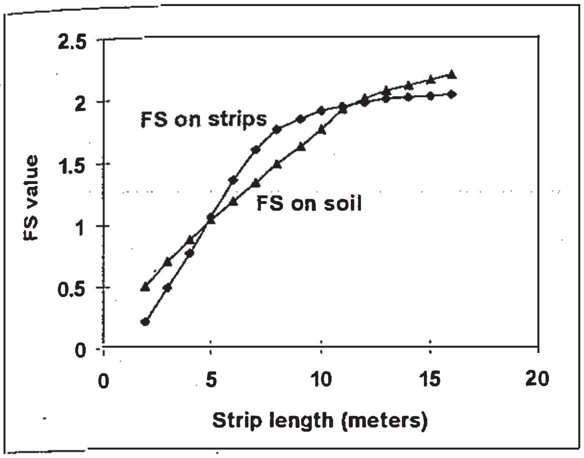

Fig.3 FS values for two definitions of safety

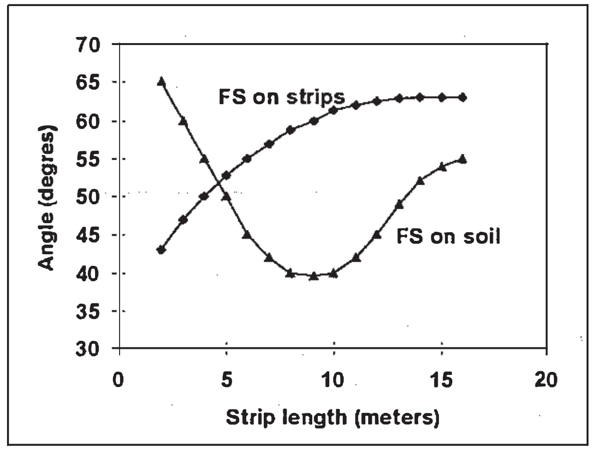

Fig.4 Critical angle α for two definition of safety

This discrepancy in FS values depending on the safety definition is shown in Fig. 3. FS has been computed for a vertical wall (9 meter high) with a given strip arrangement and a rectangular cross section with only the strip length as a variable. It is apparent that whether safety is taken on the soil (as in 1. above) or on the strips (as in 2.) the same coefficient will not be achieved. If a coefficient of 1.5 is required, one computation will lead to 6.5 meters long strips and the other to 8 meters. This is more than 20% difference!

Fig. 4 presents the values of α_1 for the critical geometry which is also very different for the two safety definitions.

The only case when the two approaches will lead to the same values of FS and α_1 is the limit equilibrium when FS = 1.

It should be noted that this problem occurs with the same discrepancies when applying the global equilibrium method (BASTICK, 1991).

4. LIMIT STATE EQUILIBRIUM

Since the equilibrium is mechanically independent on the way the coefficient of safety is defined all methods will have in common that :

a : for a given geometry $\{O;\alpha_1;\alpha_2\}$ if equilibrium is just achieved λ or FS = 1.00, whatever the definition may be.

b : if λ or $FS(O;\alpha_1;\alpha_2) > 1$ (respectively $< 1.$) for one definition, it will also be larger (respectively less) than 1 for other definitions.

c : a: and b: applied for various sets of geometries $(O;\alpha_1;\alpha_2)$ will lead to the same being true for FS (the safety coefficient for the structure, i.e. the minimum λ value). However, if FS is not unity as said above, FS will depend on the definition. In addition the critical geometry $\{O;\alpha_1;\alpha_2\}$ which would correspond to the critical rupture surface in a slope stability problem, will also depend on the definition chosen for FS.

The above observations are a good ground to favour the limit state design with load factors and partial safety coefficients. Indeed, once the desired factors and coefficients have been applied, the verification that the required safety is achieved implies a comparison of the reduced or resulting FS (or more precisely ODF [Over Design Factor]) with unity (ODF > 1.00). Under these circumstances, the way ODF is computed has no more importance.

The second method being the easiest approach from a computational point of view may be used.

In the end the computation is carried out in the following way :

1. Factor the driving forces : these include principally the soil unit weight, the traffic surcharge load and other loads such as point or localised surcharge and seismic force. Permanent driving forces are factored with a coefficient around 1.2 to 1.5 depending on the type of load and on the local code.

2. Reduce the mechanical strength of all the materials by applying a factor of safety and, when applicable, rules to take durability into account. Soil friction will be represented by tanϕ, where ϕ is the internal angle of friction. Interaction phenomenon should also be divided by a specific coefficient of safety. These coefficients will vary according to countries and type of material from 1.2 to 1.6 typically.

3. Once the factored loads and reduced mechanical characteristic values have been obtained the system should be proved to be stable (λ or ODF larger than unity) for all possible geometries of the double wedges.

5. JUSTIFICATION WITH DOUBLE WEDGE

Using the above guide lines, it is possible to write a program which will check that a given structure verifies the safety criteria of a given code. In our case we have chosen the load factors and safety factors of the French norm NF P 94-220 (1992).

5.1 Comparison with coherent gravity method

In order to compare the two methods, we designed with the coherent gravity method a large number of walls of different heights and shapes and computed their overdesign factors (O.D.F.) using the double wedge analysis. In all cases, ODF values were above unity, which indicates a more conservative approach from the coherent gravity method than from the double wedge analysis. This was usually more apparent for low walls with longer strips.

Figure 5 presents the ODF values of 9 meter high walls with a rectangular cross section as a function of their strip length:

- for very long strips (15 to 8 meters) ODF values are around 1.5 to 2 indicating the coherent gravity method to be, in this case, highly conservative as compared with the double wedge approach.
- when strips are shortened down to around 0.5H, minimum admissible value according to the code, ODF decreases to around 1. It is interesting to note that the coherent gravity method calls for more than 4 strips exactly at the point when the ODF of a wall with 4 strips per panel would fall below unity.
- finally, attempts to use very short strips with the coherent gravity method quickly leads to unrealistic numbers of strips corresponding to large values of double wedge ODF.

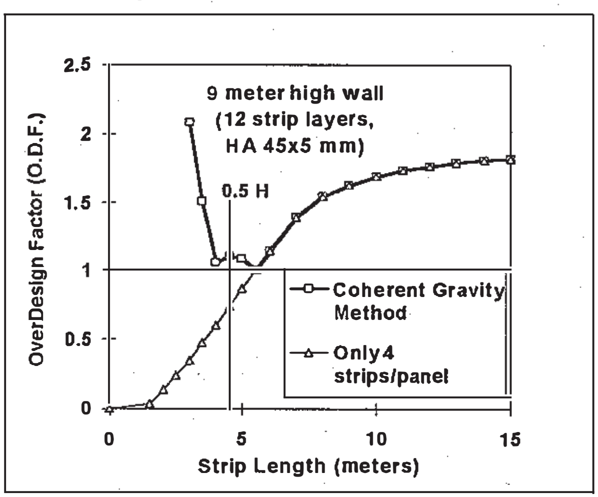

Fig.5 Double wedge ODF for structures designed with coherent gravity method

We attribute these differences between the methods to the way technological minima are taken into account:

- when using a coherent gravity method the number of unit strips per panel is rounded off for each layer. Statistically, this leads to ½ strip too much for each panel. Since a practical number of strips is around 4 to 8, this corresponds to 6 to 15% overdesign at each level, hence for the whole structure. Obviously for low walls, where the minimum number of strips is already too much, the overdesign will be even more. The same applies for very long strips.
- when using a global stability method or the double wedge method this effect does not appear.

Another important point is that the double wedge method leads to no limitation of shape while coherent gravity method is clearly not suited for cross sectional shapes far out of the code scope (e.g. not applicable to very short strips). This is added freedom and an advantage of the double wedge method, but it may lead to dangerous design in the hands of inexperienced designers.

5.2 Comparison with slip circle method

A series of wall with height varying from 3 to 12 meters and strip length varying from 4 to 8 meters was designed in accordance with the code and then checked with either the double-wedge method or with the TALREN slip circle program (Schlosser, 1984). In order to be consistent, limit state design with load factors and partial safety factors was used for the slip circle program.

It is interesting to note that, for all the walls, the following results were obtained :

- the position of the critical slip circle from TALREN was very close to the broken line formed by the double wedge
- the values of the ODF were very close to one an other, especially when they were close to unity. (For most of the example test ODF varied from 0.9 to 1.3). In nearly all the computed instances the ratio $ODF_{TALREN}/ODF_{DOUBLE\text{-}WEDGE}$ is within the range 0.96 to 1.02.
- for most of the cases ODF_{TALREN} is slightly lower than $ODF_{DOUBLE\text{-}WEDGE}$, which would indicate slip circles methods to be more conservative than the double wedge. However, it is possible by choosing particular values for lower strip resistance to build cases where the opposite would be true.

In the end, one can conclude that there is quite a good agreement between slip circle analysis and double wedge method.

5.3 Comparison for minimalized structure

In order to get rid of the technological overdesign associated with using an integer number of available strips we further made comparison with *minimalized structures.* We call *minimalized structure* a structure computed with the coherent gravity method where **for each level**:

1 : the strip width has been reduced so that the overdesign against a strip slippage (tf/tm) is just unity

2 :the strip resistance has been reduced so that the overdesign in tensile resistance (tr/tm) is just unity.

Such a structure would have an ODF equal to unity for all strip layers.

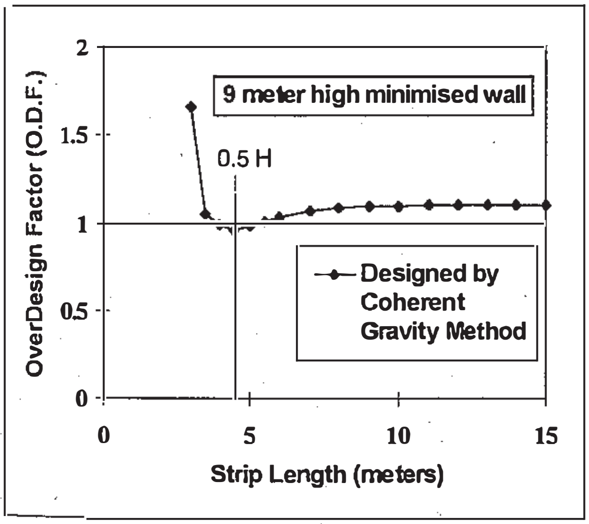

Fig.6 Double wedge ODF for minmalised structure

Figure 6 presents the overdesign factor computed with the double wedge method for such minimalised stuctures:

- as can be seen within the validity area of the coherent gravity method (i.e. with strips longer than 0.5 H) the $ODF_{DOUBLE\text{-}WEDGE}$ is very close to unity which confirms the good agreement of the two methods.

- for very short strips $ODF_{DOUBLE\text{-}WEDGE}$ becomes very large as was the case in fig. 5.

- for strips in a small range around 0.5 H, the coherent gravity method is slightly less conservative than the double wedge, but as can be seen on fig. 5, this is more than compensated by the technological constraints of placing an integer number of strips per panel.

- for strips longer than 0.7 H the coherent gravity method will lead to 5 to 10% more strips even before the technological rounding off.

6. DESIGN VERSUS JUSTIFICATION

Up to this point the double wedge concept can only be used for checking a given wall but not to construct a solution to a given problem as is the case with the coherent gravity method. However, by stepping the pivot from top to bottom, and inputting the minimum required amount of strip at each level it is possible to turn the method into a design method which will construct a strip arrangement. The program is modified in the following way:

1 : The pivot is placed just below the first strip layer to define its resistance and width.

2 : Assuming the resistance and width computed above for the first strip layer, the pivot is positioned just below the second strip layer to define the second strip layer resistance and width.

3 : the same is done downward till all the strip layers are defined.

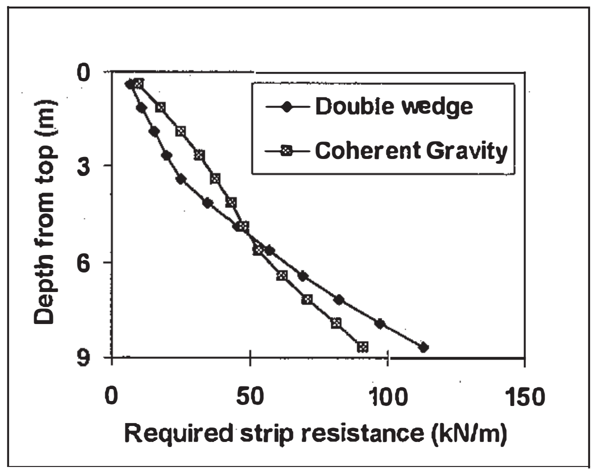

Fig.7 Design comparison for strip resistance

Figure 7 presents the required strip resistance as a function of depth for designs done using the coherent gravity and the double wedge methods. Whilst we know from fig. 6 that the total amount of strip required will be nearly the same for the two methods, it is apparent that the distribution is not quite the same :

- near the top, coherent gravity method requires more resistance than double wedge. This can be attributed to the fact that the double wedge method does not account for the locked-in stress effects. We know that compaction results in higher horizontal stresses (or a near K_o state of stress) which cannot be represented with the double wedge method.

- in the lower part of the wall this effect is counterbalanced by larger required resistance fo the double wedge method.

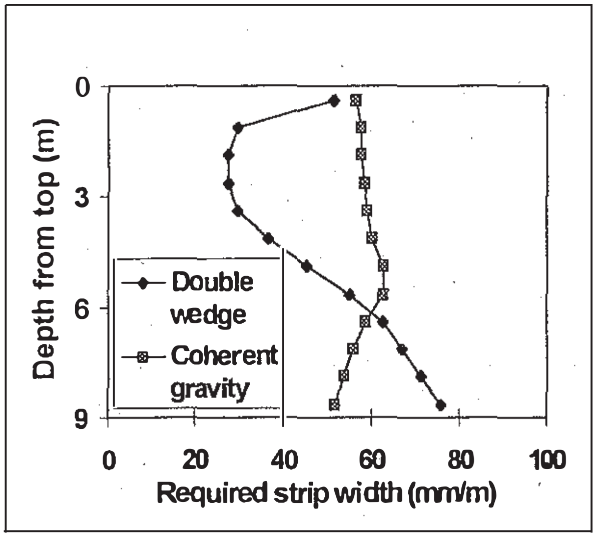

Fig.8 Design comparison for strip friction

Figure 8 presents the required friction capacity or strip width for the two methods. If we except layer 1, the double wedge method calls for a strip width more or less proportional to depth. This is quite different from the coherent gravity method where strip width requirement is nearly constant throughout the structure. We interpret this difference by the fact that the double wedge analysis is just an equilibrium method that cannot take into account deformations of the structure. Hence the top of the structure, with this approach, is totally independent of what lies beneath it. However, small movements and deformations will appear in a real structure, which will transmit the information to the top and affect its behaviour. In the coherent gravity method, this is taken into account through the line of maximum tensile force, which depends on the global geometry of the structure.

7. CONCLUSION

The double wedge equilibrium method should be used in conjunction with the critical state approach where loads are factored and material properties are reduced by use of partial safety factors. Attempts to define a global factor of safety will lead to meaningless results from a physical point of view and the critical wedge will be undefined.

Under these restrictions the double wedge method may be used for controlling the design of a structure and shows good agreements with codified methods such as the coherent gravity method.

We also tried to turn the method into a design method to construct a solution for a mechanically stabilised earth fill problem. This works from a theoretical point of view but the strip distribution obtained is very different from the usual one. Hence the double wedge method is really better suited as a verification method rather than as a proper design method

REFERENCES

French Ministry of Transport 1979 Recommendations and Rules of the Art for Reinforced Earth Structures

AFNOR, 1992.NF P94-220:Soil Reinforcement -- Backfilled structures reinfoced with quasi-inextensible and flexible reinforcing strips or sheets – Design

Bishop, A. W., 1955,The use of the Slip Circle in the Stability Analysis of slopes, *Geotechnique*, London, England, Vol.5, No.1, pp.7-17.

Taylor,D.W.,1937 Stability of Earth Slopes, *Journal of the Boston Society of Civil Engineers*, Vol.24, No.3, pp.337-386.

Schlosser, F., Blondeau, F., Guilloux, A, Christiansen, M. 1984. Talren: méthode de calcul des ouvrages en terre renforcée, C.R. cooloque int. Renforcement en place des sols et des roches, pp 219-224, Paris.

Schlosser, F., Unterreiner, P., Plumelle, C. 1993 Validation of calculation methods of soil nailed walls based on the experimentation of the National Project CLOUTERRE, *Rev. Française de Géotechnique*, No 64, pp.1-20.

Bastick, M. 1991, Calculs aux états limites en géotechnique, le cas de la Terre Armée, *Jounées du CFMS*, St Rémy-lès-Cheuvreuses, France.

Gässler, G. 1988, Soil-nailing - Theoretical basis and practical design, *Internat. Geotech. Sympos. on Theory and Practice of Earth Reinforc.*, Fukuoka, Japan.